Automatic
Proofs and Refutations
in Isabelle/HOL

A Survey

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Isabelle

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- Interactive theorem prover

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- Based on higher-order logic

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The user perspective:

- Frequently fast and effective
- Sometimes annoyingly incomplete

Metis

by Joe Hurd for HOL4

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- `algebra`: Gröbner basis

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Sledgehammer

by Paulson, Meng, Susanto, Quigley (at Cambridge)
Wenzel, Immler, Meyer, Blanchette (at Munich)

How  works

higher-order, typed

Isabelle

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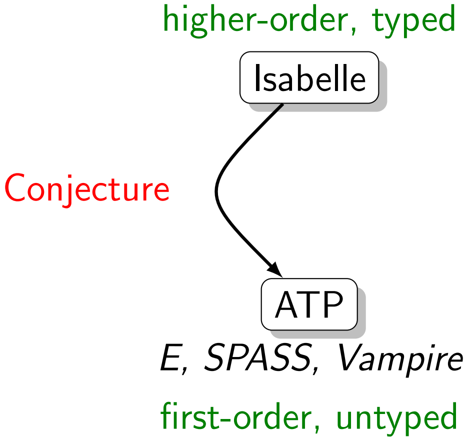
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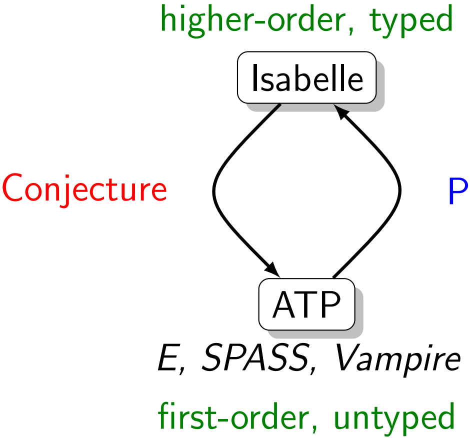
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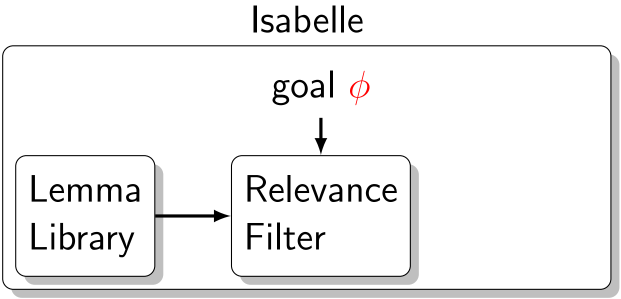

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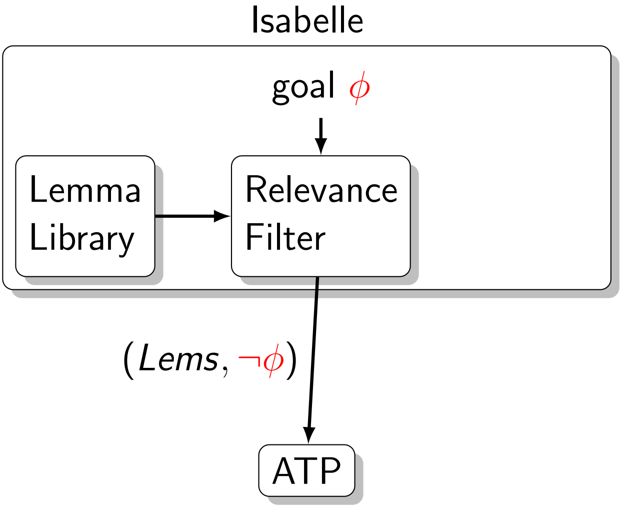

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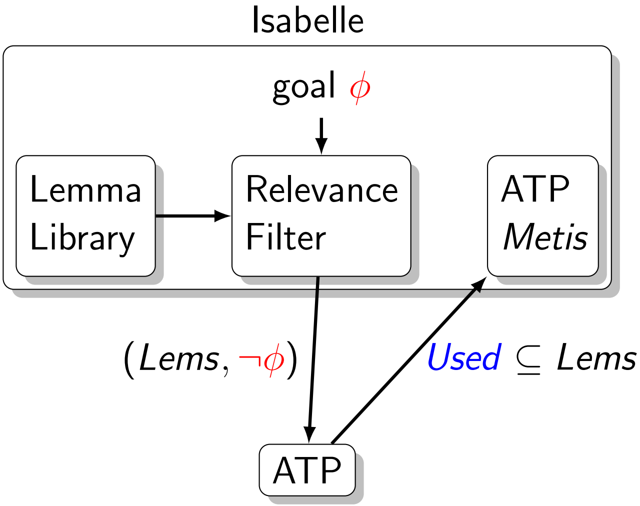
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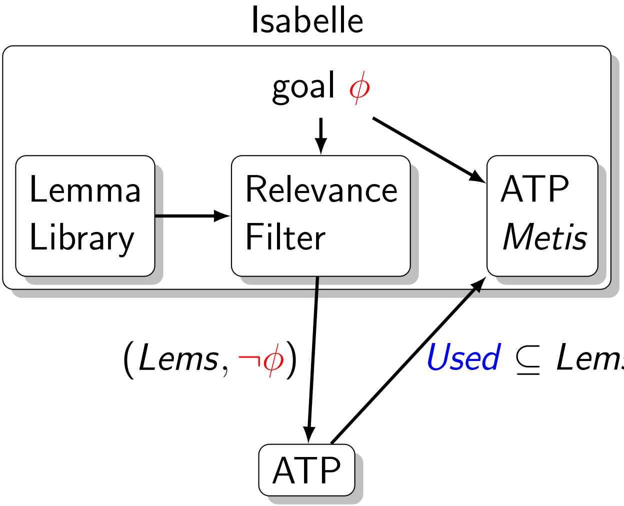
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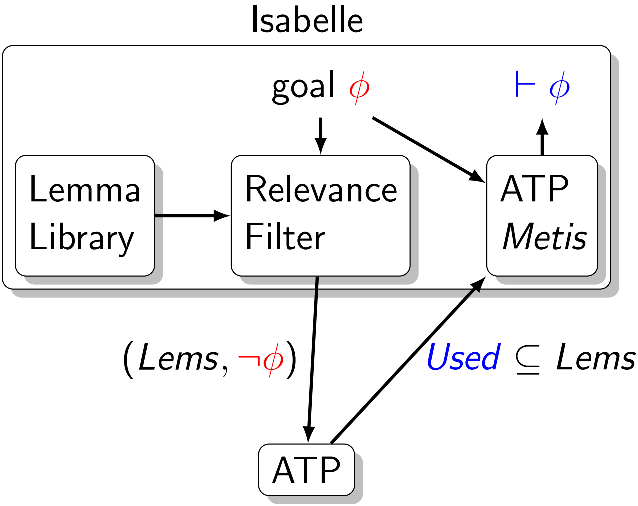

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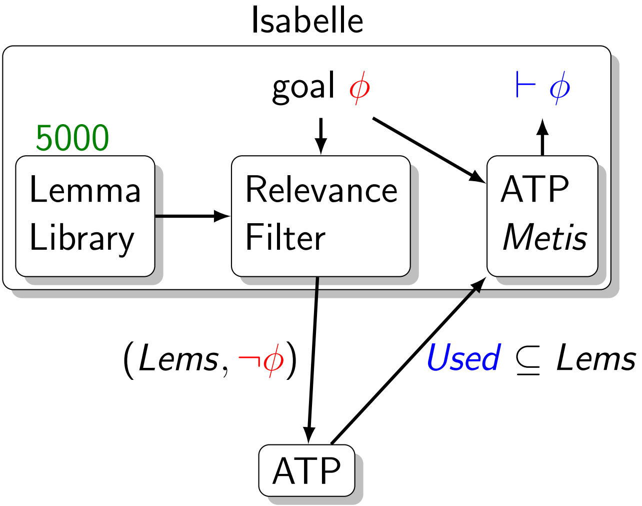
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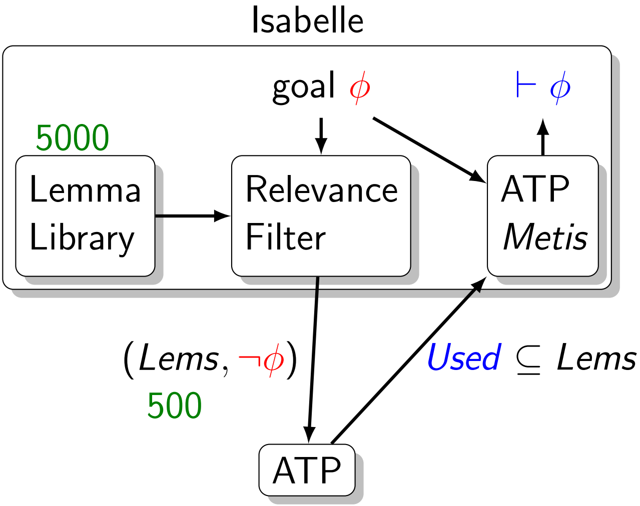
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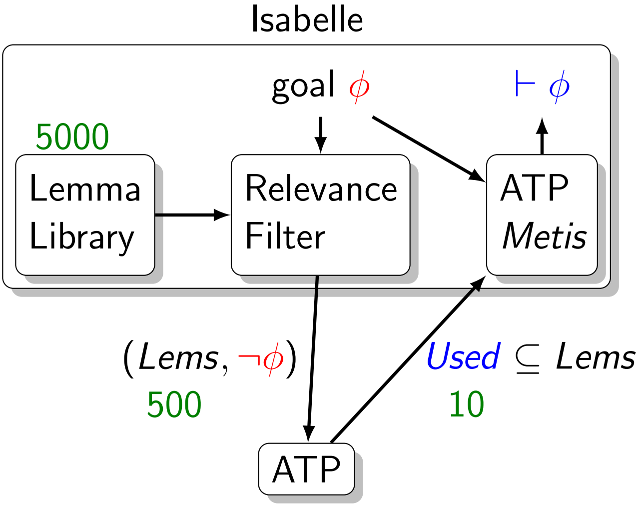
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External ATPs act as relevance filter for Metis

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  { **assume** *xs* $\neq$ []
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Work in progress!

Sledgehammer: empirical evaluation

Based on 1200 goals from diverse theories covering

- arithmetic
- inductive datatypes
- recursive functions
- inductive definitions
- set theory

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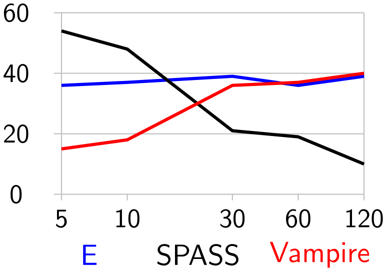

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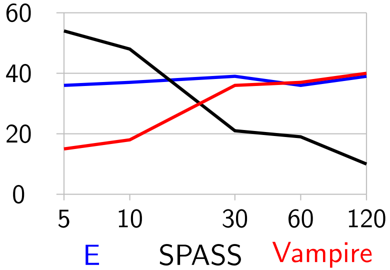

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[Böhme, Nipkow, IJCAR 2010]

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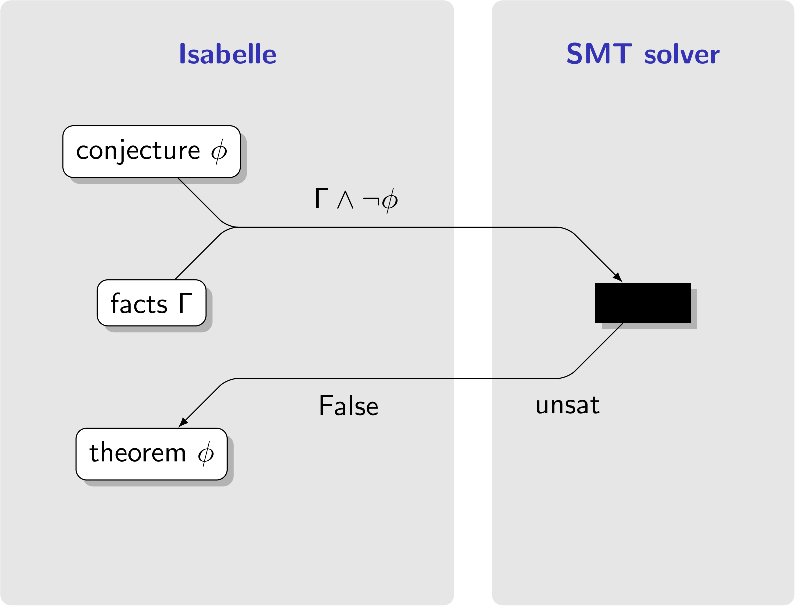

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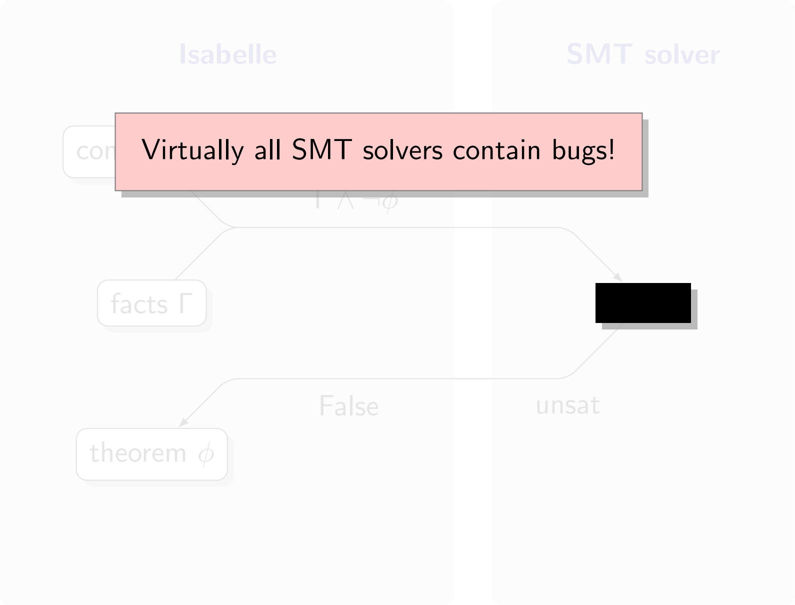

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- leading SMT solver
- developed by Microsoft Research
- supports proofs

Z3

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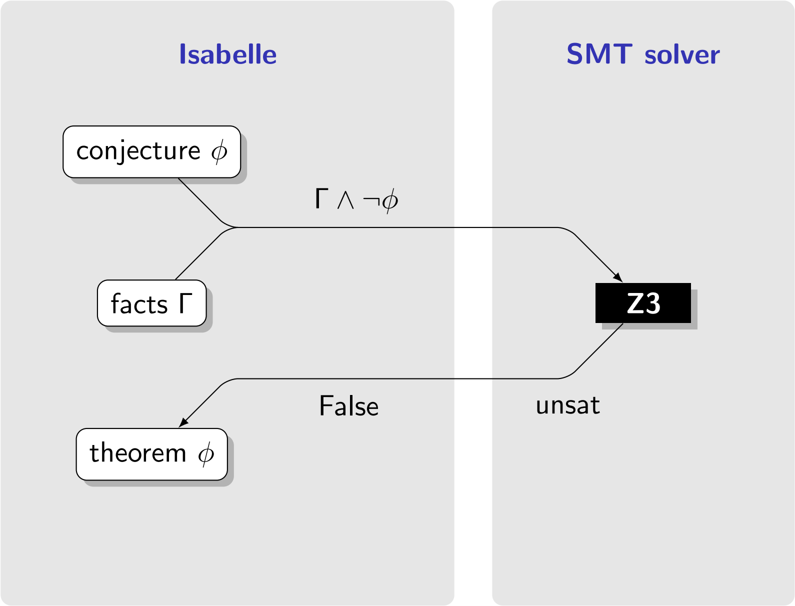

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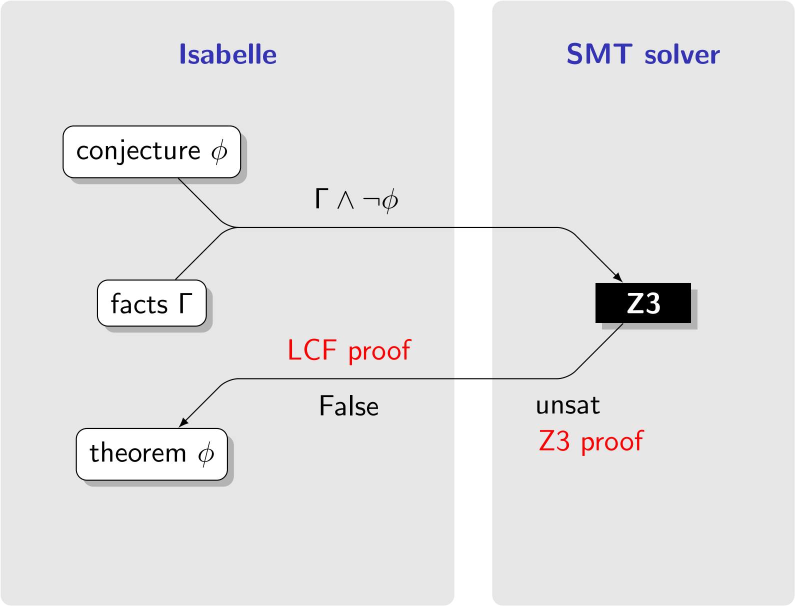

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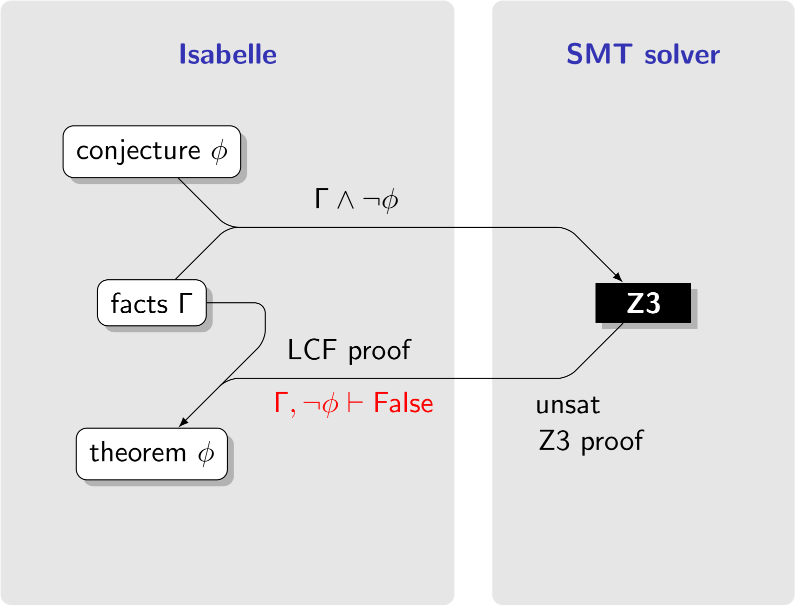

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Theory rules are lacking valuable information!

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[Böhme, Weber, ITP2010]

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Profiling:

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- 50% of the runtime is spent on 15% of all Z3 proof steps

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Example: $x^2 + y^2 + z^2 = 1 \Longrightarrow (x + y + z)^2 \leq 3$

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Approaches for counterexample generation

Nitpick: search for finite countermodels of formula using external tools (SAT solvers)

- Covers most of HOL, including non-executable constructs (e.g. quantifiers)
- Slow for complex data structures

Quickcheck: evaluate formula on random values for free variables using code generator

- Fast
- Restricted to executable fragment of HOL

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[Blanchette, Nipkow, ITP-10]

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On average, Nitpick falsifies ≈42% of all mutants

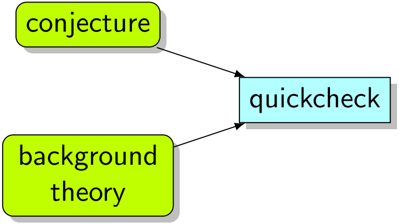
1. Automatic Proof
   - Internal Provers
   - External Provers
     - FO ATPs
     - SMT Solvers
     - SDP Solvers
2. Automatic Refutation
   - Nitpick
   - Quickcheck

Quickcheck

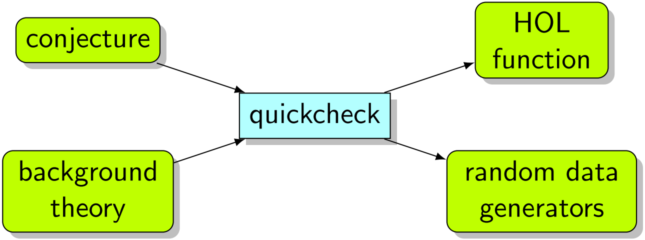
conjecture

background theory

Quickcheck



Quickcheck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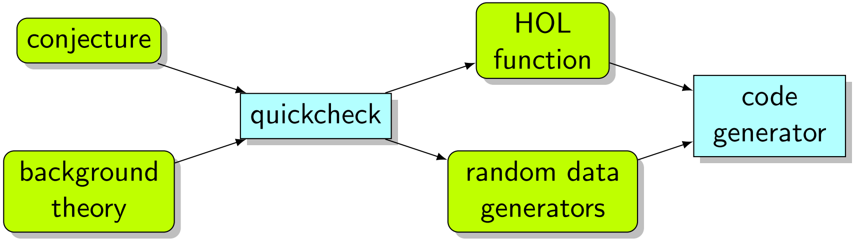
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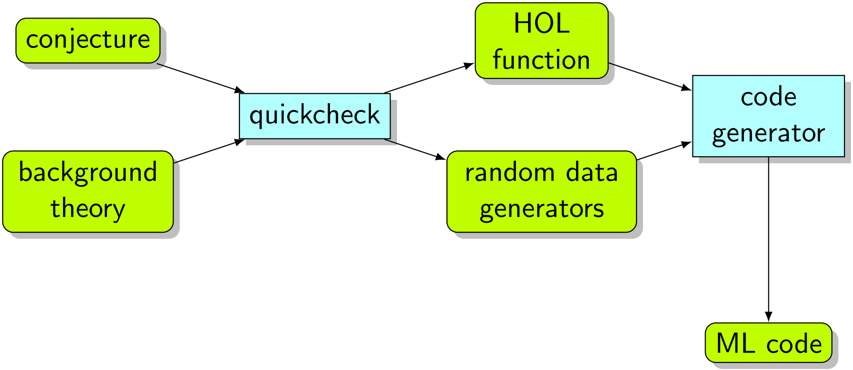
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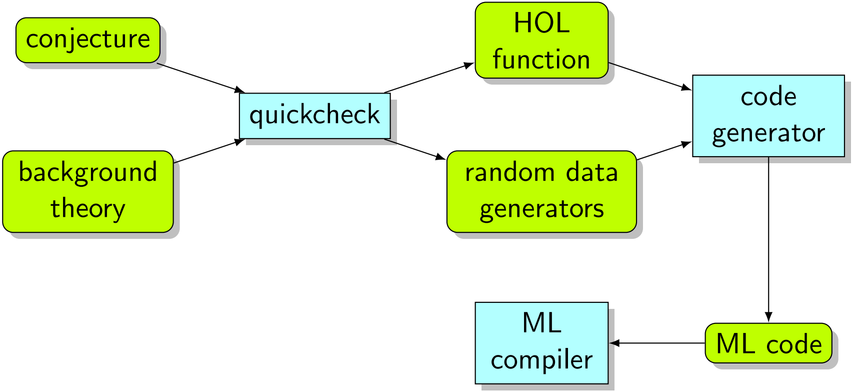

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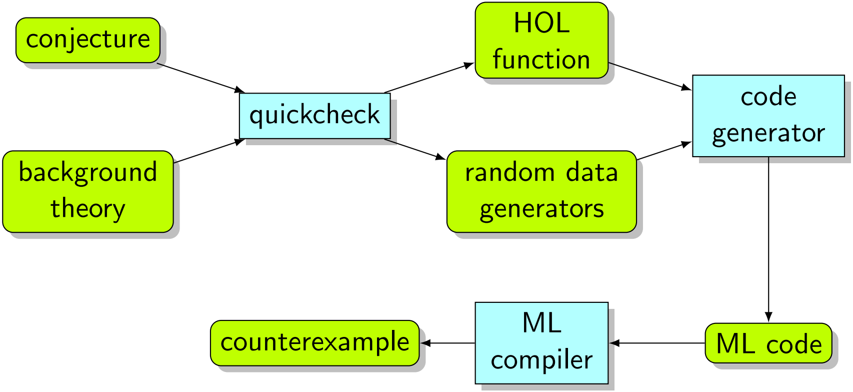

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Current and future work

Focus: Sledgehammer

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