Quantitative Verification and Synthesis using MDPs with Ratio Objectives

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UJF Grenoble/Verimag

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Outline

Quantitative Verification and Synthesis

- Verification: Does the system fulfill the specification?

Quantitative Verification and Synthesis

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- Synthesis (1): Give me a system fulfilling the specification

Quantitative Verification and Synthesis

- Verification: Does the system fulfill the specification?
- Synthesis (1): Give me a system fulfilling the specification
- Synthesis (2): Give me the *best* system fulfilling the specification

Controlling Pumps

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What to Control? (Environment)

- Modes: Off/Slow/Fast

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- May break down

What to Control? (Environment)

- Modes: Off/Slow/Fast
- May break down
- Repair pump (or not)

Chance (Environment)

- Pumps behave randomly

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- We model chance with probabilities

Chance (Environment)

- Pumps behave randomly
- We model chance with probabilities
- Example: Fast pumps wear out sooner

How to Evaluate? (Specification)

- Waterflow

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- Waterflow
- Repair costs

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- Waterflow
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- Repair bulk discount

How to Evaluate? (Specification)

- Waterflow
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- No pump running $\Longrightarrow$ penalty

How to Evaluate? (Specification)

- Waterflow
- Repair costs
- Repair bulk discount
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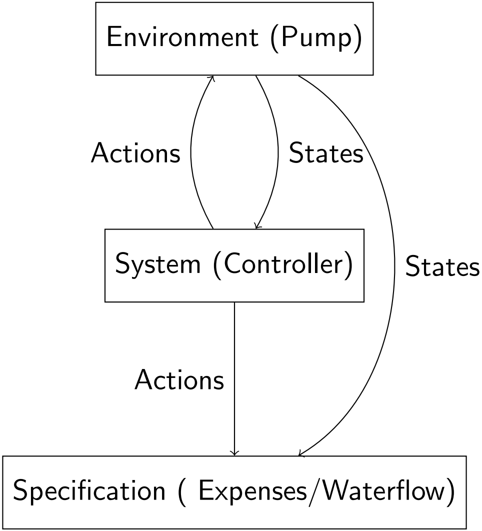
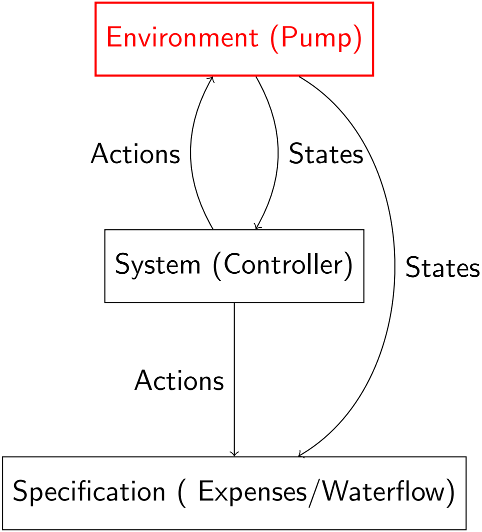
Aim of the controller: Minimize $\frac{Expenses}{Waterflow} \frac{\$}{m^3}$

Overview


Environment (Pump)
Actions
States
System (Controller)
States
Actions
Specification (Expenses/Waterflow)

Outline

Overview



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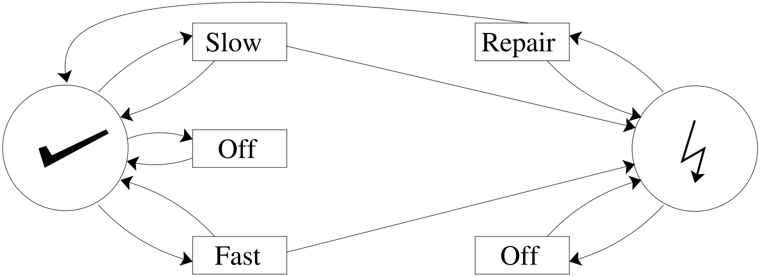
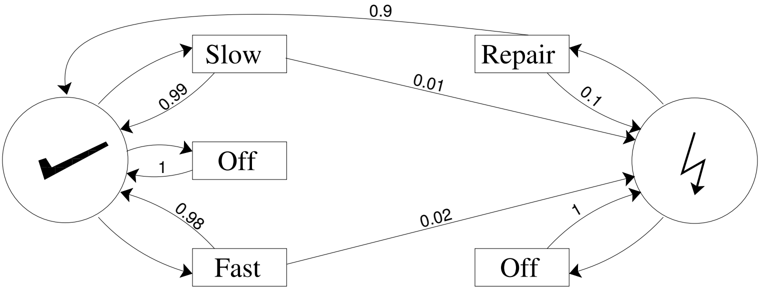
Modeling a Pump (Environment)



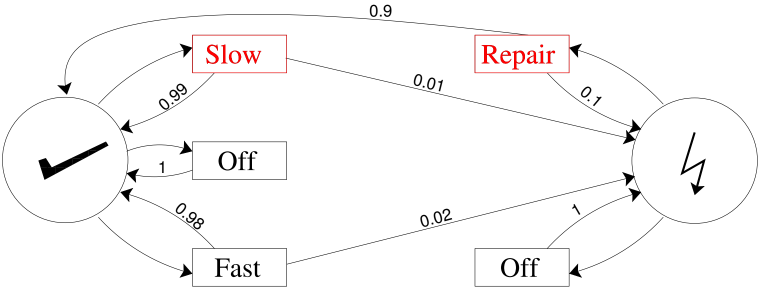
Modeling a Pump (Environment)



Markov Decision Process (MDP)

Modeling a Pump (Environment)

Strategy: Decide what action to takes when



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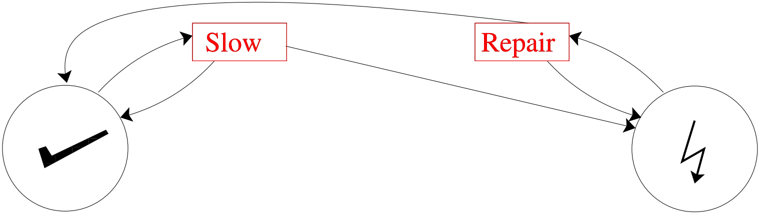
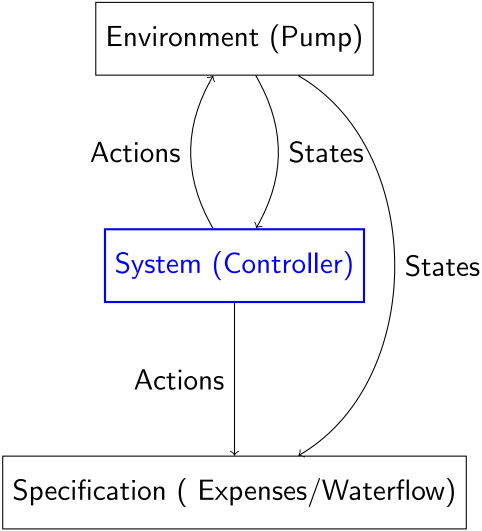


Markov Chain

Overview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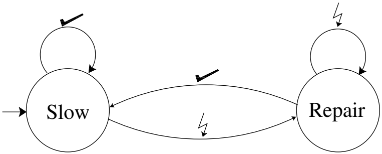
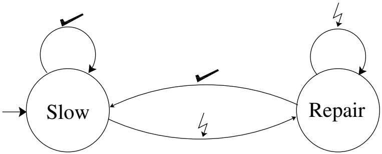
Modeling Systems (Controllers)



Example: Controlling a single pump

- Finite-state machine

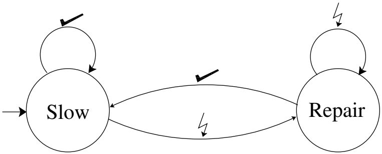
Modeling Systems (Controllers)



Example: Controlling a single pump

- Finite-state machine
- Input: MDP states

Modeling Systems (Controllers)



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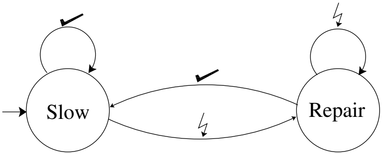

Example: Controlling a single pump

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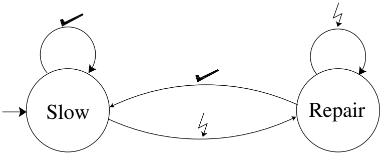
- Output defined by state (Moore Machine)

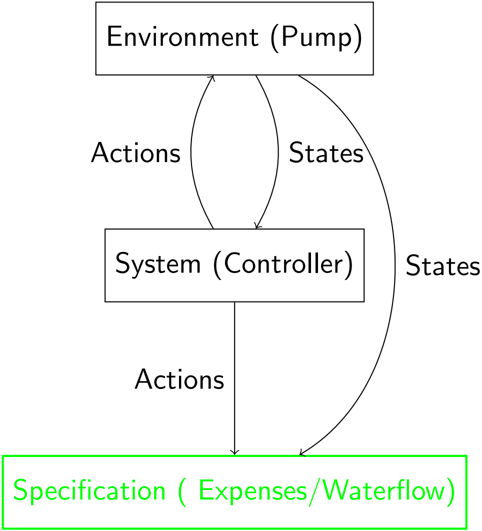
Modeling Systems (Controllers)



Example: Controlling a single pump

- Finite-state machine
- Input: MDP states
- Output: Actions
- Output defined by state (Moore Machine)
- System is a strategy

Overview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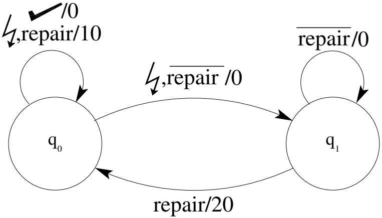
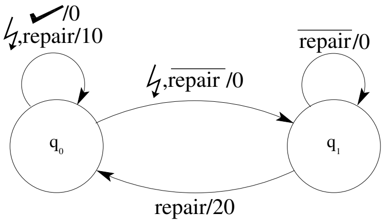
Evaluating Systems (Specification)



- Automaton with numbers on transitions

Evaluating Systems (Specification)



- Automaton with numbers on transitions
- Input: MDP states and actions

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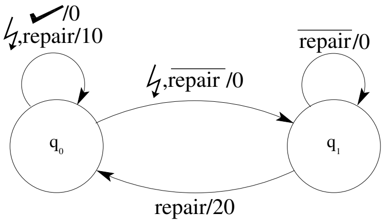

- Automaton with numbers on transitions
- Input: MDP states and actions
- Numbers represent how good or bad a decision is

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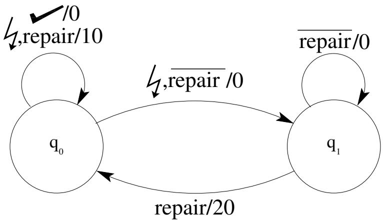

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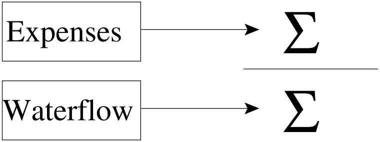
Minimize



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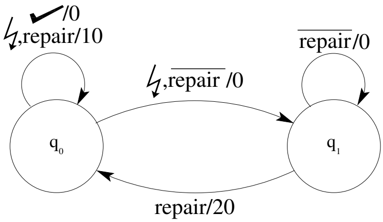

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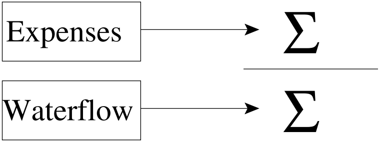
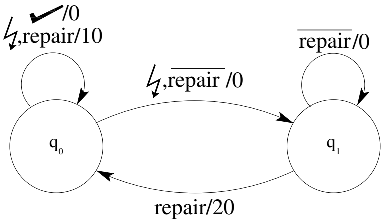


$$= \frac{\sum_{i=l}^{m} c_1(s_i,a_i)}{1+\sum_{i=l}^{m} c_2(s_i,a_i)}$$

Evaluating Systems (Specification)



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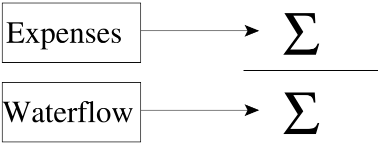

$$= \quad \liminf_{m\to\infty} \frac{\sum_{i=l}^{m} c_1(s_i,a_i)}{1+\sum_{i=l}^{m} c_2(s_i,a_i)}$$

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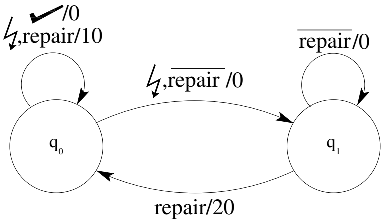

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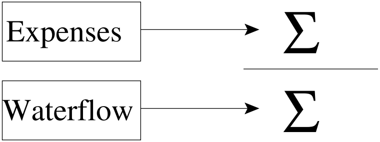

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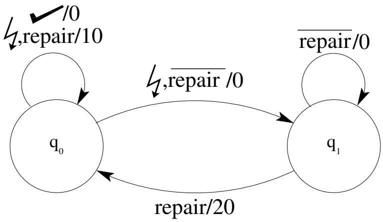

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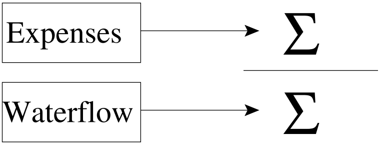

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Specification for Multiple Pumps

- Parallel product

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- Parallel product
- Water costs: sum output

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- Water costs: sum output
- Repair costs: maximum

Specification for Multiple Pumps

- Parallel product
- Water costs: sum output
- Repair costs: maximum
- Additional costs if no pump is running

Outline

Main Goals

1. Evaluate the quality of a controller (Verification)
2. Automatically find the best controller (Synthesis)

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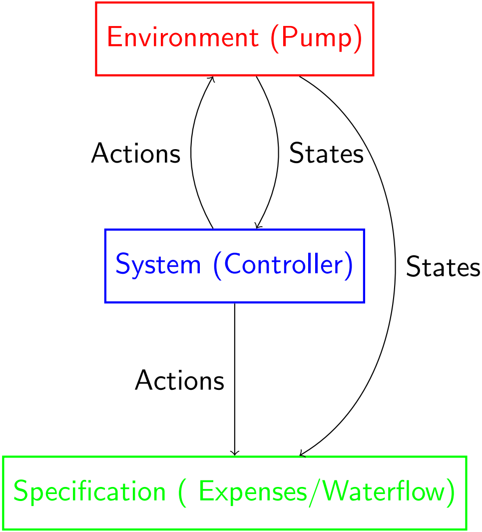
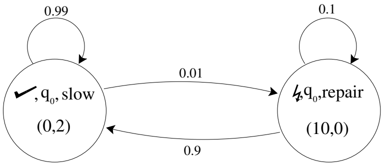
Overview (for Synthesis)



Verification

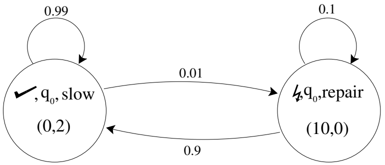
MDP with cost $\times$ controller $=$ Markov Chain



$$\mathbb{E}(\lim \lim\inf \frac{\sum_{i=l}^{m} c_1(s_i)}{1+\sum_{i=l}^{m} c_2(s_i)})$$

Verification

MDP with cost $\times$ controller $=$ Markov Chain



$$\mathbb{E}(\lim \lim\inf \frac{\sum_{i=l}^{m} c_1(s_i)}{1+\sum_{i=l}^{m} c_2(s_i)})$$

$$= \frac{\sum_s c_1(s)\nu(s)}{1+\sum_s c_2(s)\nu(s)}$$

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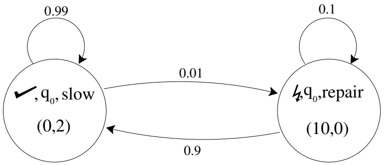
MDP with cost $\times$ controller $=$ Markov Chain



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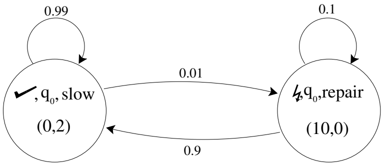
- Calculate ν by linear equations (stationary distribution)

Verification

MDP with cost $\times$ controller $=$ Markov Chain



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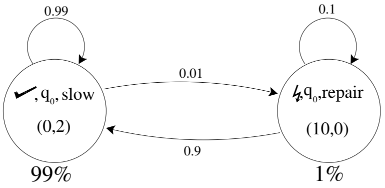
- $\frac{0.01\times 10}{0.99\times 2}$

Verification

MDP with cost $\times$ controller $=$ Markov Chain



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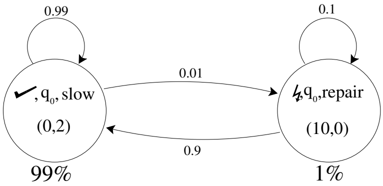
- Calculate ν by linear equations (stationary distribution)
- $\frac{0.01\times 10}{0.99\times 2}$
- Corner case: Value infinity (all denominators zero)

Verification

MDP with cost × controller = Markov Chain



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- Calculate ν by linear equations (stationary distribution)
- $\frac{0.01\times 10}{0.99\times 2}$
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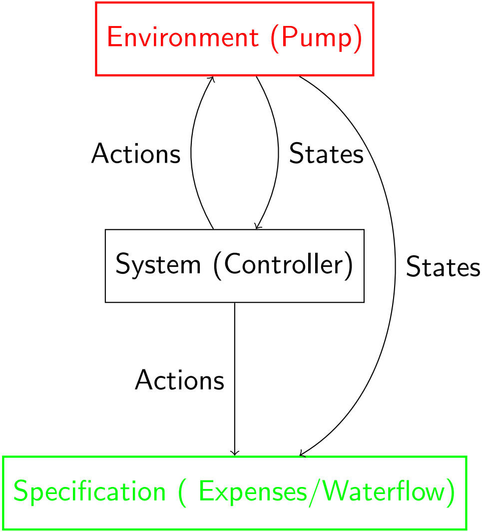
For non-SCC MC: SCCs, then probability of reaching these components

Main Goals

1. Evaluate the quality of a controller (Verification)
2. Automatically find the best controller (Synthesis)

Overview (for Verif. and Synthesis)



(One) Algorithm

How to find an optimal strategy?

- $x(s,a)$ Prob. of being in s and choosing a

$\forall s\ \forall a\ x(s,a)$

Minimize $\frac{\sum x(s,a) c_1(s,a)}{\sum x(s,a) c_2(s,a)}$

subject to

1. $\sum_{s,a} x(s,a) = 1$
2. $\forall s\ \sum_a x(s,a) = \sum_{s',a} x(s',a) \cdot p(s',a,s)$

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- $x(s,a)$ Prob. of being in s and choosing a
- Probability of being in s: $\sum x(s,a)$

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Minimize $\frac{\sum x(s,a) c_1(s,a)}{\sum x(s,a) c_2(s,a)}$

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- $x(s,a)$ Prob. of being in s and choosing a
- Probability of being in s: $\sum x(s,a)$
- $x(s,a)$ are a steady state distribution

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- $x(s,a)$ Prob. of being in s and choosing a
- Probability of being in s: $\sum x(s,a)$
- $x(s,a)$ are a steady state distribution
- We search for the best steady state distribution

(One) Algorithm

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- First equation: $x(s,a)$ has to be a probability distribution

(One) Algorithm

How to find an optimal strategy?

$\forall s\ \forall a\ x(s,a)$

Minimize $\frac{\sum x(s,a) c_1(s,a)}{\sum x(s,a) c_2(s,a)}$

subject to

1. $\sum_{s,a} x(s,a) = 1$
2. $\forall s\ \sum_a x(s,a) = \sum_{s',a} x(s',a) \cdot p(s',a,s)$

- $x(s,a)$ Prob. of being in s and choosing a
- Probability of being in s: $\sum x(s,a)$
- $x(s,a)$ are a steady state distribution
- We search for the best steady state distribution
- First equation: $x(s,a)$ has to be a probability distribution
- Second equation:
  - $\sum_{s',a} x(s',a) \cdot p(s',a,s)$: Prob. of going to s

Outline

Experimental Results (Behavior)

How smart are our controllers?
2 Pumps:

- Play it safe
- Turn one pump on in fast mode.
- When one pump is broken, repair it, turn the other one on in slow mode.

Experimental Results (Behavior)

How smart are our controllers?
3 Pumps:

- Turn all pumps on fast
- When one pump is broken, turn one on fast, the other two off
- When two pumps are broken, turn the last one on fast, repair the rest

Experimental Results (Behavior)

How smart are our controllers?
4 Pumps:

- Turn all pumps on fast
- When one pump is broken, turn two pumps on fast, the other two off
- When two pumps are broken, turn one on fast
- When three pumps are broken, turn the last one on fast, repair the rest

Thank you

Fin.