Automata-Theoretic Model Checking of Reactive Systems

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Ensuring Correctness of Hw/Sw Systems

- Uses logic to specify correctness properties, e.g.:
  - *the program never crashes*
  - *the program always terminates*
  - *every request to the server is eventually answered*
  - *the output of the tree balancing function is a tree, provided the input is also a tree ...*
- Given a logical specification, we can do either:
  - VERIFICATION: prove that a given system satisfies the specification
  - SYNTHESIS: build a system that satisfies the specification

Approaches to Verification

- THEOREM PROVING: reduce the verification problem to the satisfiability of a logical formula (entailment) and invoke an off-the-shelf theorem prover to solve the latter
  - Floyd-Hoare checking of pre-, post-conditions and invariants
  - Certification and Proof-Carrying Code
- MODEL CHECKING: enumerate the states of the system and check that the transition system satisfies the property
  - explicit-state model checking (SPIN)
  - symbolic model checking (SMV)
- COMBINED METHODS:
  - static analysis (ASTREE)
  - predicate abstraction (SLAM, BLAST)

Model Checking Real-Life Systems

- MODEL EXTRACTION:
  - give precise semantics (meaning) to what the system does and how it does it
  - the result is a (possibly infinite) directed graph in which the nodes denote states and the edges denote transitions
  - the model is an abstraction of the original system, i.e. it has more behaviors
- MODEL VERIFICATION:
  1. check whether the model satisfies a given property
  2. if no error was found, stop and report OK
  3. otherwise, check if the error is feasible in the original system
     - if yes, report ERROR
     - otherwise, refine the abstraction, by excluding the spurious behavior and goto 1

Safety vs. Liveness

- Safety : *something bad never happens*

  A counterexample is an finite execution leading to something bad happening

  Example: the program does not dereference any null pointers

- Liveness : *something good eventually happens*

  A counterexample is an infinite execution on which nothing good happens

  Example: the function terminates on any given input

Modeling Systems

Systems Dichotomies

- Deterministic/Non-deterministic
- Sequential/Concurrent
  - synchronous/asynchronous communication between processes
- Hardware/Software/Embedded
  - Hw is always finite-state (boolean data)
  - Sw is considered infinite (integers, recursive data structures, etc.)
- Transformational/Reactive
  - a transformational system takes input, computes output and stops
  - a reactive system interacts continuously with the environment

Problems in Systems Modeling

- Representing states
  - local/global components
- Granularity of actions
  - what are the atomic transitions ?
- Representing concurrency
  - one transition at a time
  - coinciding transitions

Modeling States

- $V = \{x_0, x_1, x_2 \ldots\}$ is a set of variables ranging over some domain (bool,int,...)
- $\varphi(x_0, x_1, \ldots)$ is a parameterized assertion over V e.g., $x_0 < 10,\ x_1 \leq x_2 + x_3, \ldots$
- A state is an assignment of values to the variables e.g., $s(x_0) = 2,\ s(x_1) = 3,\ s(x_2) = 5, \ldots$
- $s \models \varphi$ iff φ is true under s

Atomic Transitions

- An atomic transition is a small piece of code such that no smaller piece of code is observable
- Question: is $x \leftarrow x + 1$ observable ?
- Answer1: yes, if x is a register and the transition is executed using an `inc` machine command
- Answer2: yes, if x is variable local to a process, which is not visible to other processes

```
             int a = 0;

P1: load R1, a          P2: load R2, a
    inc R1                  inc R2
    store R1, a             store R2, a
```

Modeling Atomic Transitions

- Each transition $G \rightarrow A$ has two parts:
  - the guard G: the enabling condition
  - the action A: a multiple assignment
  - the guard and action are executed in one atomic step
- **Example**: $x > y \rightarrow x' = y \ \wedge \ y' = x$
- **Frame rule**: if a variable v' does not appear in A then implicitly $v' = v$

Initial Conditions

- $V = \{x_0, x_1, x_2, \ldots\}$ are program variables
- The initial condition is an assumption $\psi(x_0, x_1, \ldots)$
- The program can start in any state s such that $s \models \psi$
- **Example**: $x = 0, x > 0$, ...

Sequential Systems

- $V = \{x_0, x_1, x_2, \ldots\}$
- $P = \langle V, T, I \rangle$, where
  - T is a set of transitions $G \rightarrow A$ involving V
  - I is an initial condition over I
- **Example**:

$$P = \langle \{x, y\}, \{True \rightarrow x' = x + y, y > 0 \rightarrow y' = y - 1\}, x = 0 \wedge y > 0 \rangle$$

- **State space**:

$$\begin{array}{cccccc} (0,3) & (3,2) & (5,1) & (6,0) & & \\ (0,4) & (4,3) & (7,2) & (9,1) & (10,0) & \\ (0,5) & (5,4) & (9,3) & (12,2) & (14,1) & (15,0) \end{array}$$

$$\ldots$$

Concurrent Systems: the interleaving model

- $S = \langle P_1, P_2, \ldots, P_n \rangle$, where $P_i = \langle V_i, T_i, I_i \rangle$, $i = 1, \ldots, n$
- $V = \bigcap_{i=1}^{n} V_i$ are called global variables
- $L_i = V_i \setminus V$ are called local variables
- An execution is a possibly infinite sequence of states $s_0, s_1, s_2, \ldots$ such that:
  - $s_0 \models I_1 \wedge \ldots \wedge I_n$
  - for each $i = 0, 1, 2, \ldots$ there exists $j \in \{1, \ldots, n\}$ and $G \rightarrow A \in T_j$ such that $s_i \models G$ and $s_i, s_{i+1} \models A$ (s_i is the valuation of unprimed and s_{i+1} the valuation of primed variables)
  - i.e., exactly one process is executed at the time
  - the frame rule applies to that specific process

Mutual Exclusion Example

- $P_i = \langle \{m, x, l_i\}, \{t_1^i, t_2^i, t_3^i\}, m = 0 \wedge x = 0 \wedge l_i = 0 \rangle$, for $i = 1, 2$ where

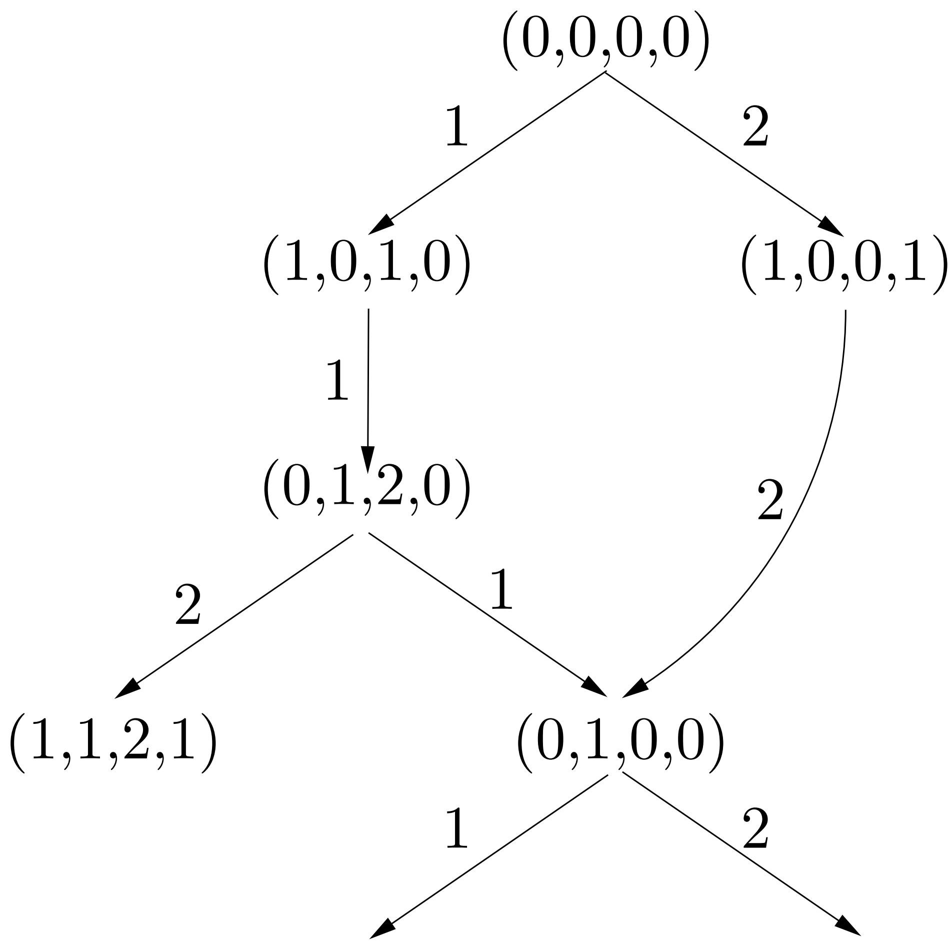
$$
\begin{array}{rcrcl}
t_1^i & : & l_i = 0 \wedge m = 0 & \rightarrow & l_i' = 1 \wedge m' = 1 \\
t_2^i & : & l_i = 1 \wedge m = 1 & \rightarrow & l_i' = 2 \wedge m' = 0 \wedge x' = x + 1 \\
t_3^i & : & l_i = 2 & \rightarrow & l_i' = 0
\end{array}
$$

- A possible execution:

$$
\begin{array}{rcccccc}
(m, x, l_1, l_2) & : & (0,0,0,0) & \xrightarrow{1} & (1,0,1,0) & \xrightarrow{1} & (0,1,2,0) \\
 & & & \xrightarrow{2} & (1,1,2,1) & \xrightarrow{2} & (0,2,2,2)
\end{array}
$$

Mutual Exclusion Example

- No deadlock: in every state there is at least one enabled transition
- Mutex: there is at most one process in the critical section at any time
- No starvation: if a process attempts to enter the critical section, then eventually it will enter
- **Future problem**: the state space is infinite!



Fairness

Global assumptions on the process scheduler:

- Weak process fairness: if some process is enabled continuously from some state, then it will be executed
- Weak transition fairness: if some transition is enabled continuously from some state, then it will be executed
- Strong process fairness: if some process is enabled infinitely often, then it will be executed
- Strong transition fairness: if some transition is enabled infinitely often, then it will be executed

Fairness Example

$$x = 0 \wedge y = 0 \wedge z = 0 \wedge l_1 = 0 \wedge l_2 = 0$$

```
P1::= 0: x'=1              P2::= 0: while y=0 do
                                 1:    z'=z+1
                                          []
                                 2:    if x=1 then y'=1
```

Does P_1 terminate ? Does P_2 terminate ?

- No fairness: nothing guaranteed
- Weak fairness: P_1 terminates
- Strong process fairness: P_1 terminates
- Strong transition fairness: both P_1 and P_2 terminate

Linear Temporal Logic

Reasoning about infinite sequences of states

- Linear Temporal Logic is interpreted on infinite sequences of states
- Each state in the sequence gives an interpretation to the atomic propositions
- Temporal operators indicate in which states a formula should be interpreted

Example 1 *Consider the sequence of states:*

$$\{p, q\}\ \{\neg p, \neg q\}\ (\{\neg p, q\}\ \{p, q\})^\omega$$

Starting from position 2*,* q *holds forever.* $\square$

Kripke Structures

Let $\mathcal{P} = \{p, q, r, \ldots\}$ be a finite alphabet of *atomic propositions*.

A *Kripke structure* is a tuple $K = \langle S, s_0, \rightarrow, L \rangle$ where:

- S is a set of *states*,
- $s_0 \in S$ a designated *initial state*,
- $\rightarrow : \ S \times S$ is a *transition relation*,
- $L : S \rightarrow 2^{\mathcal{P}}$ is a *labeling function*.

Paths in Kripke Structures

A *path* in K is an infinite sequence $\pi \;:\; s_0, s_1, s_2 \ldots$ such that, for all $i \geq 0$, we have $s_i \longrightarrow s_{i+1}$.

By $\pi(i)$ we denote the i-th state on the path.

By π_i we denote the suffix $s_i, s_{i+1}, s_{i+2} \ldots$.

$$\boxed{\inf(\pi) = \{s \in S \mid s \text{ appears infinitely often on } \pi\}}$$

If S is finite and π is infinite, then $\inf(\pi) \neq \emptyset$.

Linear Temporal Logic: Syntax

The alphabet of LTL is composed of:

- atomic proposition symbols $p, q, r, \ldots,$
- boolean connectives $\neg, \vee, \wedge, \rightarrow, \leftrightarrow,$
- temporal connectives $\bigcirc, \Box, \Diamond, \mathcal{U}, \mathcal{R}.$

The set of LTL formulae is defined inductively, as follows:

- any atomic proposition is a formula,
- if φ and ψ are formulae, then $\neg\varphi$ and $\varphi \bullet \psi$, for $\bullet \in \{\vee, \wedge, \rightarrow, \leftrightarrow\}$ are also formulae.
- if φ and ψ are formulae, then $\bigcirc\varphi$, $\Box\varphi$, $\Diamond\varphi$, $\varphi\mathcal{U}\psi$ and $\varphi\mathcal{R}\psi$ are formulae,
- nothing else is a formula.

Temporal Operators

- $\bigcirc$ is read at the next time (in the next state)
- $\Box$ is read always in the future (in all future states)
- $\Diamond$ is read eventually (in some future state)
- $\mathcal{U}$ is read until
- $\mathcal{R}$ is read releases

Linear Temporal Logic: Semantics

$$K, \pi \models p \iff p \in L(\pi(0))$$

$$K, \pi \models \neg\varphi \iff K, \pi \not\models \varphi$$

$$K, \pi \models \varphi \wedge \psi \iff K, \pi \models \varphi \text{ and } K, \pi \models \psi$$

$$K, \pi \models \bigcirc\varphi \iff K, \pi_1 \models \varphi$$

$$K, \pi \models \varphi \mathcal{U} \psi \iff \text{there exists } k \in \mathbb{N} \text{ such that } K, \pi_k \models \psi \text{ and } K, \pi_i \models \varphi \text{ for all } 0 \leq i < k$$

Derived meanings:

$$K, \pi \models \Diamond\varphi \iff K, \pi \models \top \mathcal{U} \varphi$$

$$K, \pi \models \Box\varphi \iff K, \pi \models \neg\Diamond\neg\varphi$$

$$K, \pi \models \varphi \mathcal{R} \psi \iff K, \pi \models \neg(\neg\varphi \mathcal{U} \neg\psi)$$

Examples

- p holds throughout the execution of the system (p is invariant) : $\Box p$
- whenever p holds, q is bound to hold in the future : $\Box(p \rightarrow \Diamond q)$
- p holds infinitely often : $\Box\Diamond p$
- p holds forever starting from a certain point in the future : $\Diamond\Box p$
- $\Box(p \rightarrow \bigcirc(\neg q \mathcal{U} r))$ holds in all sequences such that if p is true in a state, then q remains false from the next state and until the first state where r is true, which must occur.
- $p \mathcal{R} q$: q is true unless this obligation is released by p being true in a previous state.

Concurrent system specification in LTL

- Let $S = \langle P_1, \ldots, P_n \rangle$ be a concurrent system, where $P_i = \langle V_i, T_i, I_i \rangle$
- Absence of deadlock: $\Box \bigvee_{i=1}^{n} enabled(P_i)$
- Weak process fairness: $\bigwedge_{i=1}^{n} \Diamond\Box enabled(P_i) \rightarrow \Diamond execute(P_i)$
- Strong process fairness: $\bigwedge_{i=1}^{n} \Box\Diamond enabled(P_i) \rightarrow \Diamond execute(P_i)$

Conclusion of the first part

- Need a formal language (logic) to express queries about a system's behavior: deadlock freedom, absence of starvation, fairness conditions, etc.
- The global behavior of a system is modeled as a possibly infinite directed graph, whose nodes are labeled with assertions
- System executions are possibly infinite paths through this graph
- Linear Temporal Logic is a powerful language to express properties of system behaviors