Verification-Friendly Concurrent Balanced Binary Search Tree

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Motivation

- Balanced *Binary Search Tree* (BST) is an efficient data-structure for storing unique elements
    - No repetitions are allowed
- Formal verification:
    - Given a program, prove some property
    - In the tree:
        - prove that repetitions of elements cannot occur

Motivation

- Formal verification was applied to the sequential algorithm (e.g. using Isabelle [6])
- However, in a concurrent setting, formal verification is more complicated

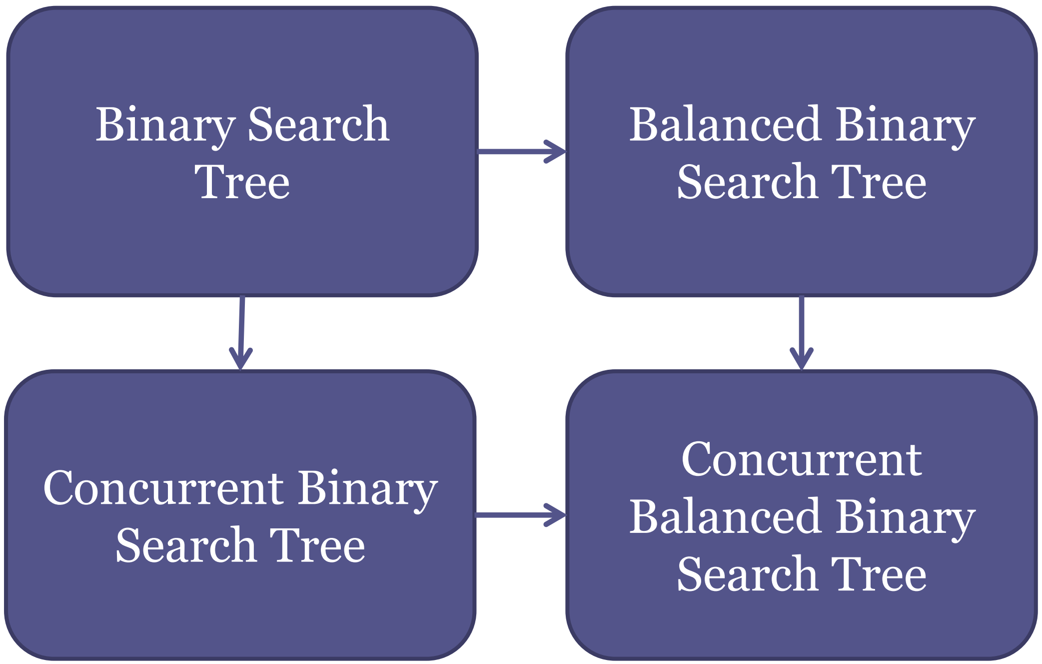
Motivation

- There seems to be a trade-off between algorithms that are easy to verify and algorithms that are practical
- A concurrent BST that is protected by a global lock is easy to verify
- Practical concurrent trees use sophisticated mechanisms
  - Many different cases to reason about
  - Harder to verify

Goal

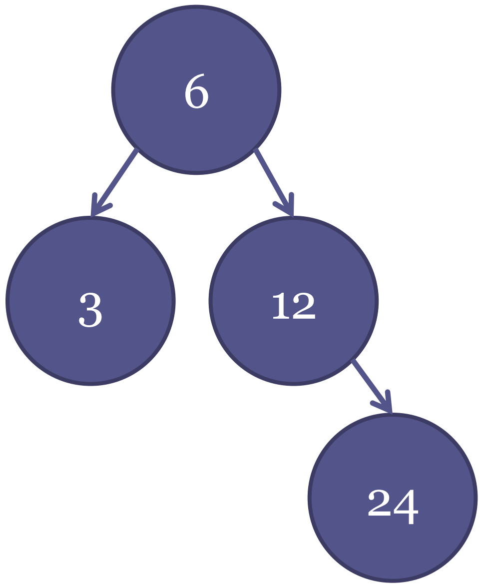
- We gap this trade-off by presenting a concurrent BST that is both practical and simple to reason about
- Our key idea:
  - Integrate the property into the algorithm
- We achieve a fine-grained locking balanced BST
- Our tree is very similar to the sequential tree
- Our mechanism allows breaking the proof into several separated proofs

Outline


Binary Search Tree
Balanced Binary Search Tree
Concurrent Binary Search Tree
Concurrent Balanced Binary Search Tree

Binary Search Tree

- A data-structure that stores elements
- Consists of nodes
- Each node represents an element
  - Internal tree
- Each element has a unique key
  - Repetitions are not allowed
- Each node in the tree holds:
  - The left sub-tree has elements with *smaller* keys
  - The right sub-tree has elements with *bigger* keys

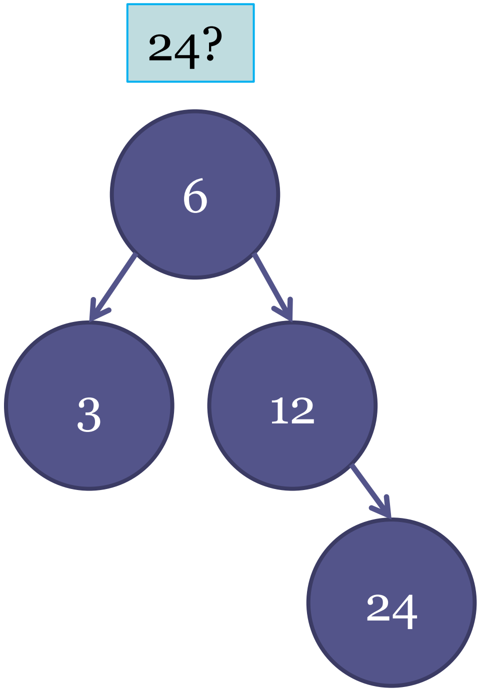


Binary Search Tree

- In other words, BST maintains two types of invariants:
  - Set invariant
    - Each key appears at most once
  - BST invariants
    - For each node:
      - The keys in the left sub-tree are smaller
      - The keys in the right sub-tree are bigger

Binary Search Tree

- Supports the following operations:
  - Contains



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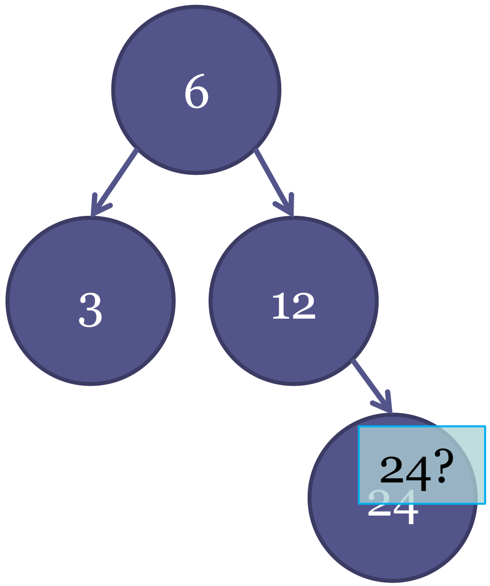
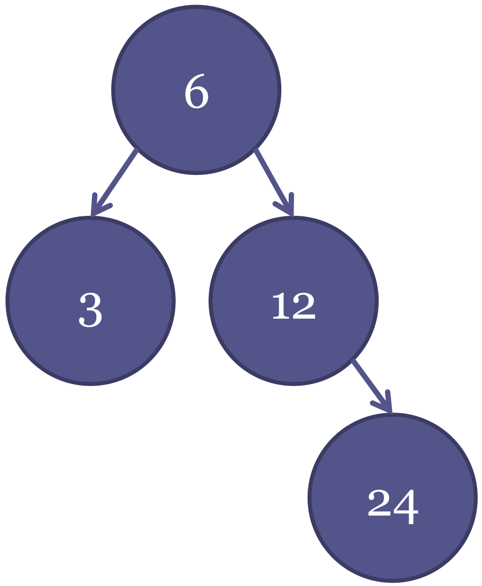


Binary Search Tree

- Supports the following operations:
  - Insert
    - The new node is always a leaf



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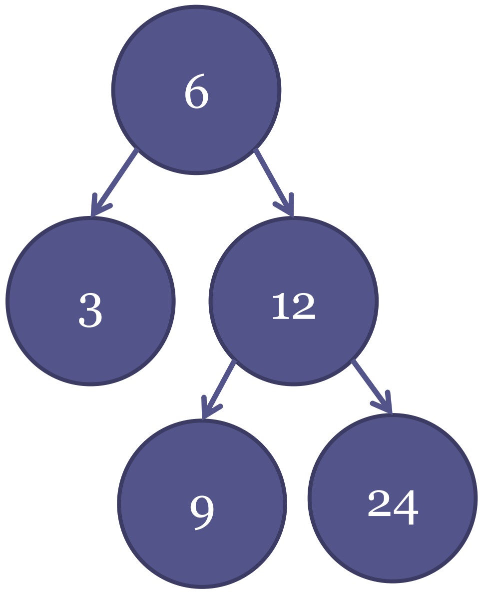
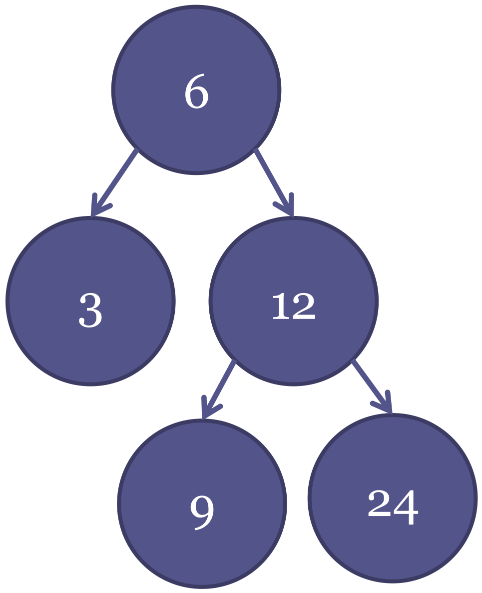


Binary Search Tree

- Supports the following operations:
  - Remove
    - The removed node, n, may be:
      - A leaf



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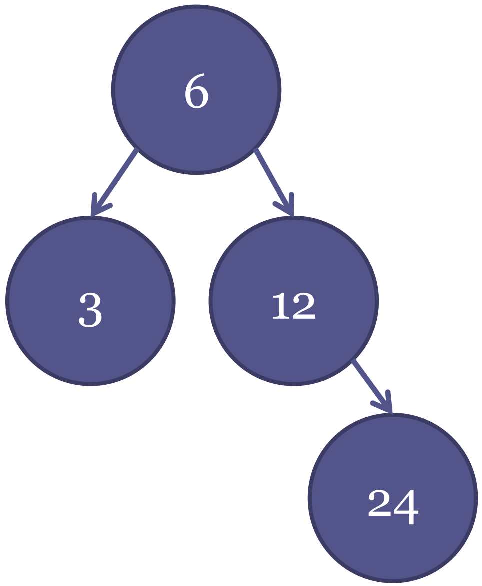
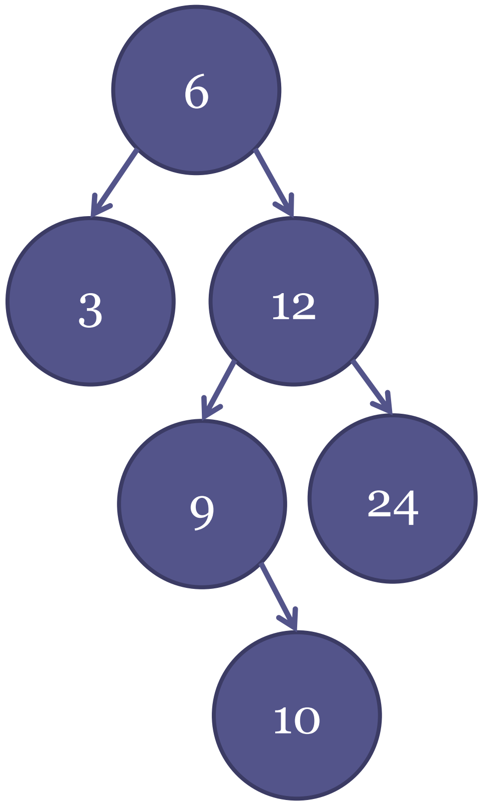


Binary Search Tree

- Supports the following operations:
  - Remove
    - The removed node, *n*, may be:
      - A leaf
      - A parent of a single child
        - *n*'s parent is connected to *n*'s child



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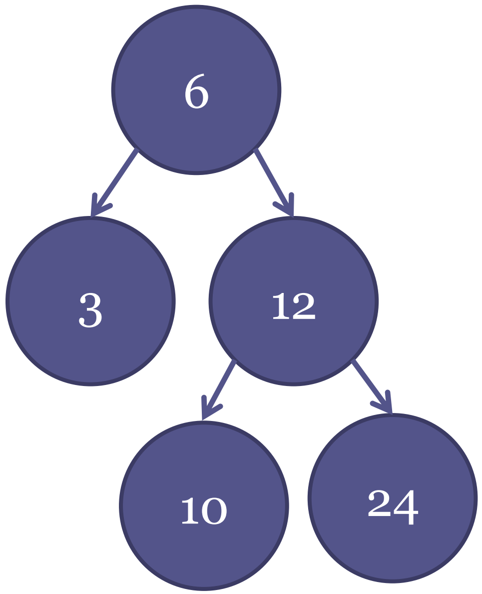

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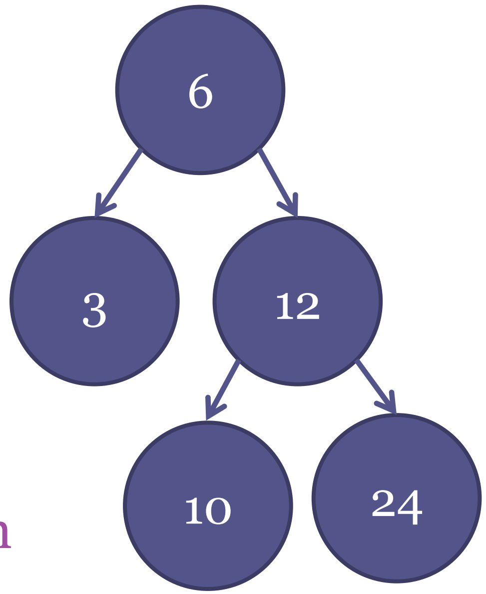

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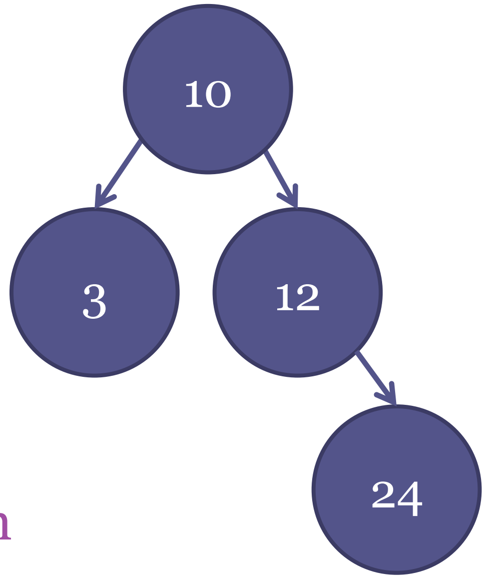

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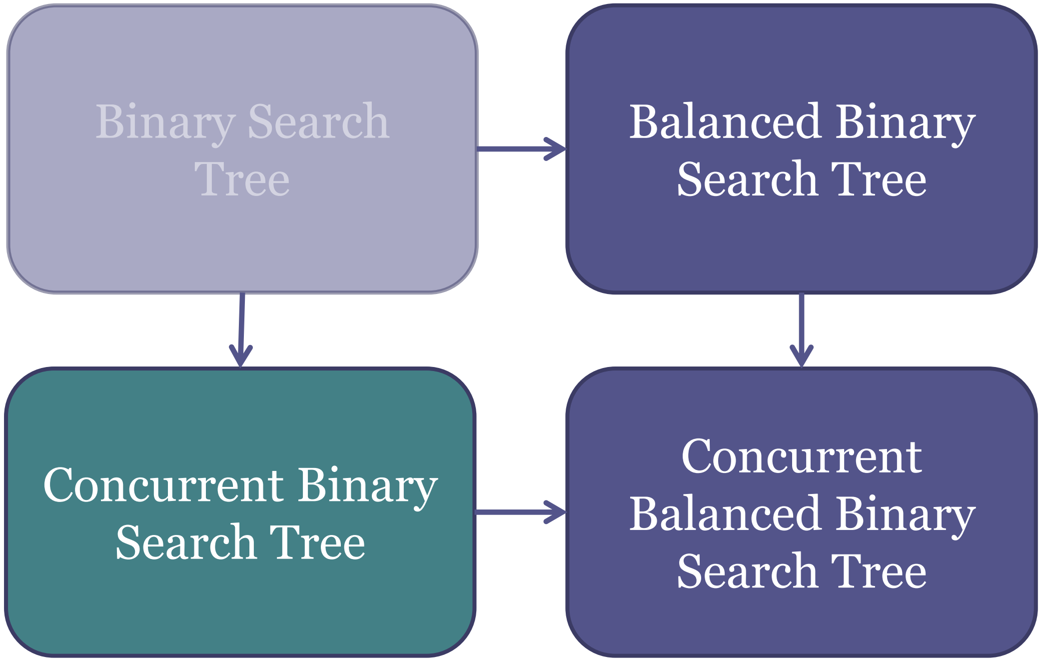
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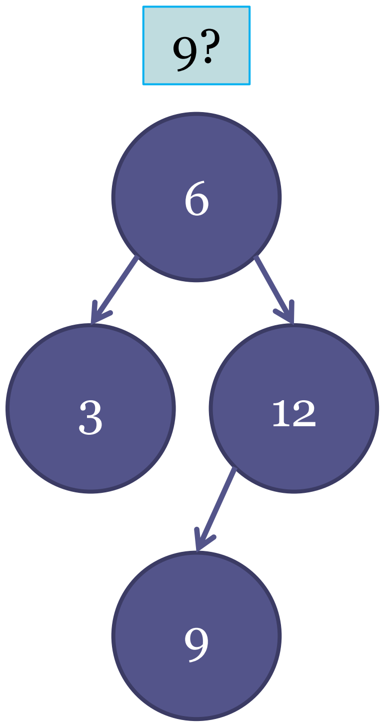
Challenges in Concurrent BST

- Consider the following tree:
  - Thread A searches for 9



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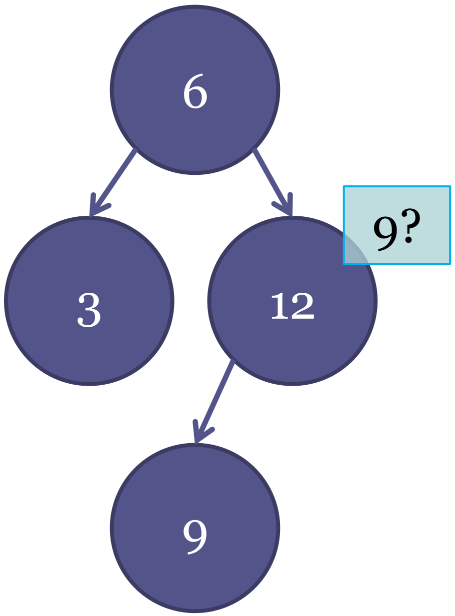
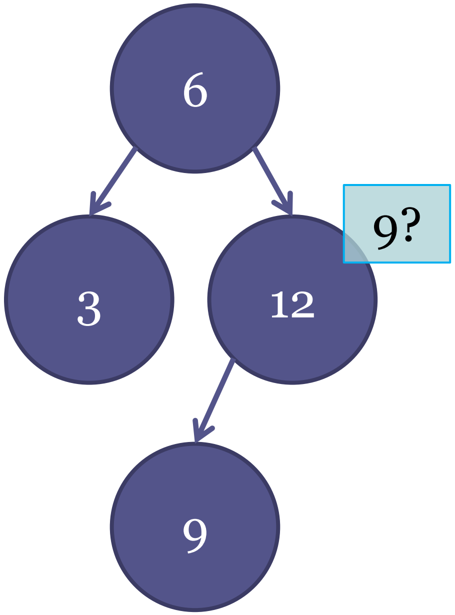


Challenges in Concurrent BST

- Consider the following tree:
  - Thread A searches for 9 and pauses
  - Thread B removes 6



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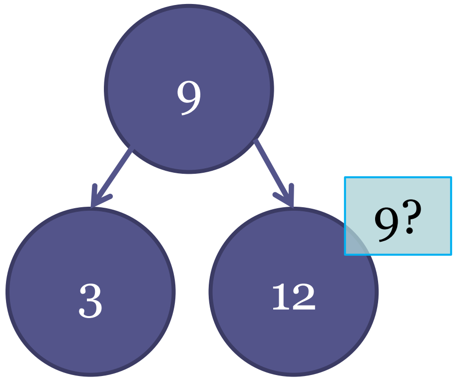

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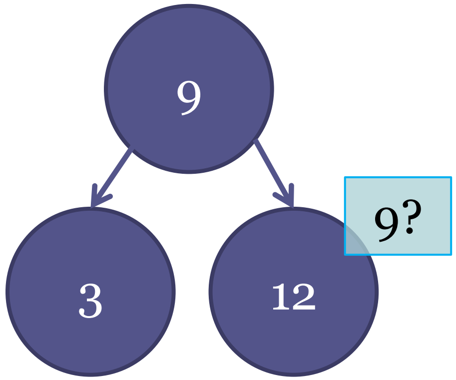
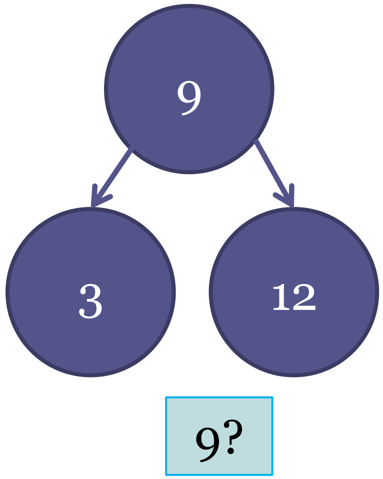


Challenges in Concurrent BST

- Consider the following tree:
  - Thread A searches for 9 and pauses
  - Thread B removes 6
  - Thread A resumes the search and observes that 9 is not present



How do others cope with this challenge?

- By not supporting the remove operation
  - Bender et al. [1]

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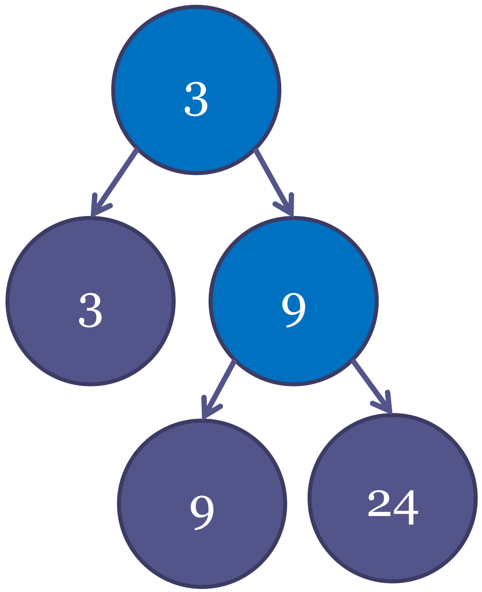
- By using external trees
  - Only leaves can be removed
  - Use more space than internal trees
  - Ellen et al. [4]



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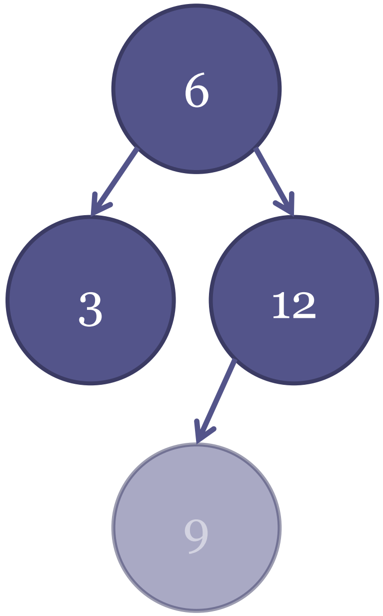
  - Marking the node as *logically* removed



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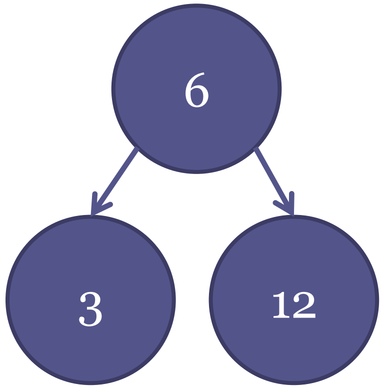
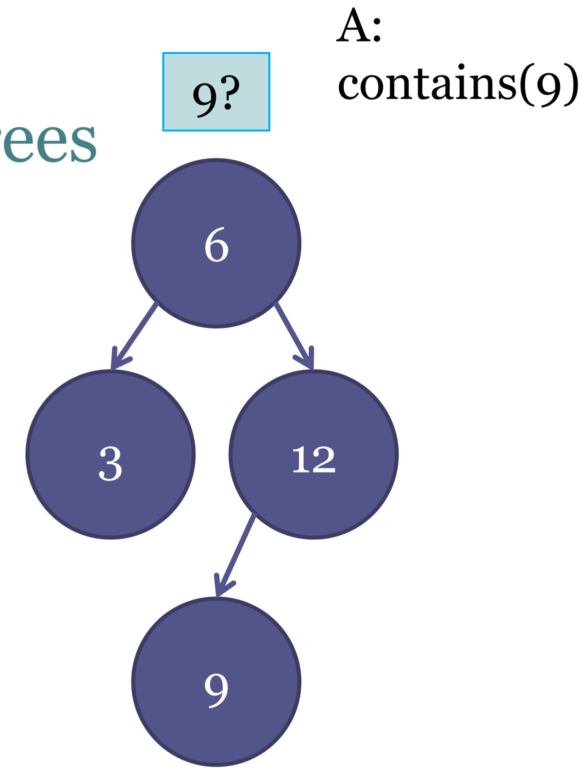
- Many concurrent algorithms for data-structures remove elements in two steps:
  - Marking the node as *logically* removed
  - Update pointers to *physically* remove the node



How do others cope with this challenge?

- By marking the node as removed without *physically* removing it
  - Also known as partially-external trees
  - Bronson et al. [2]
  - Crain et al. [3]



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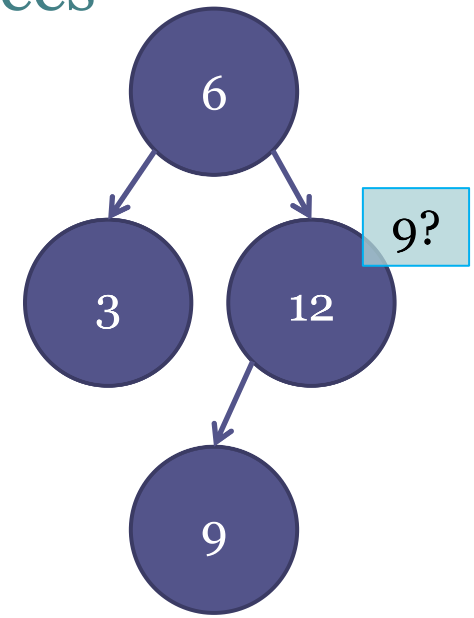

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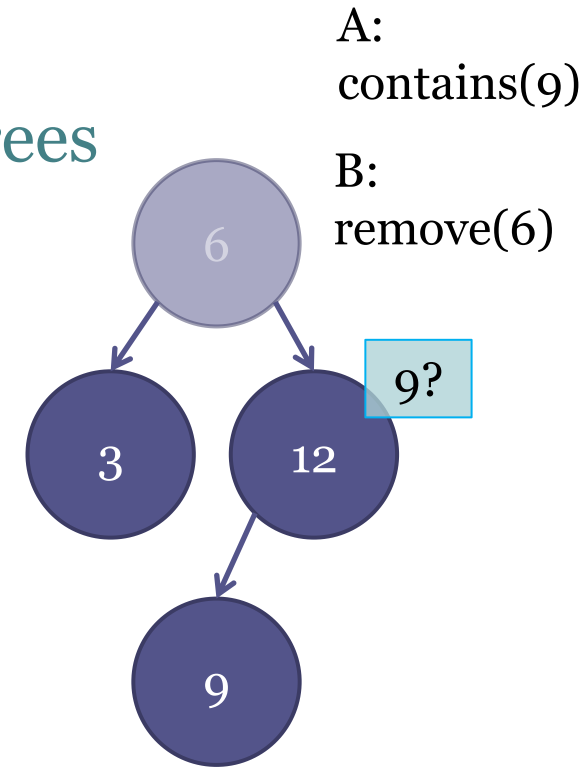

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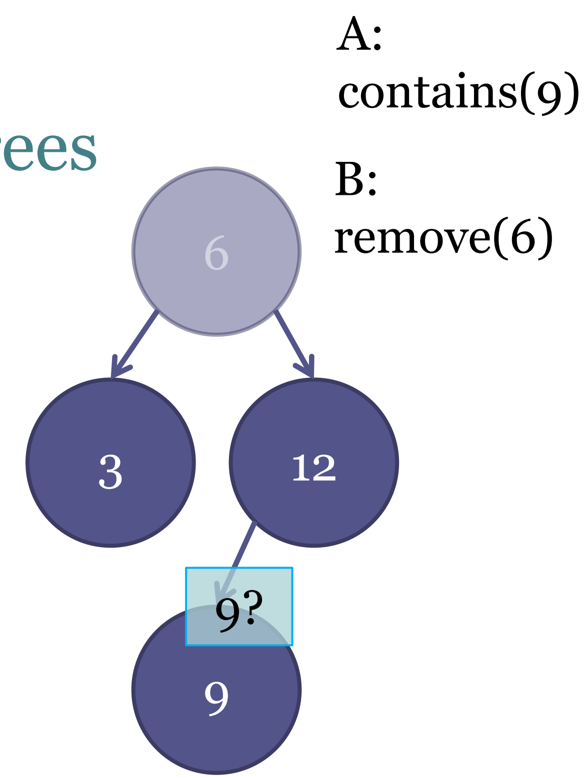

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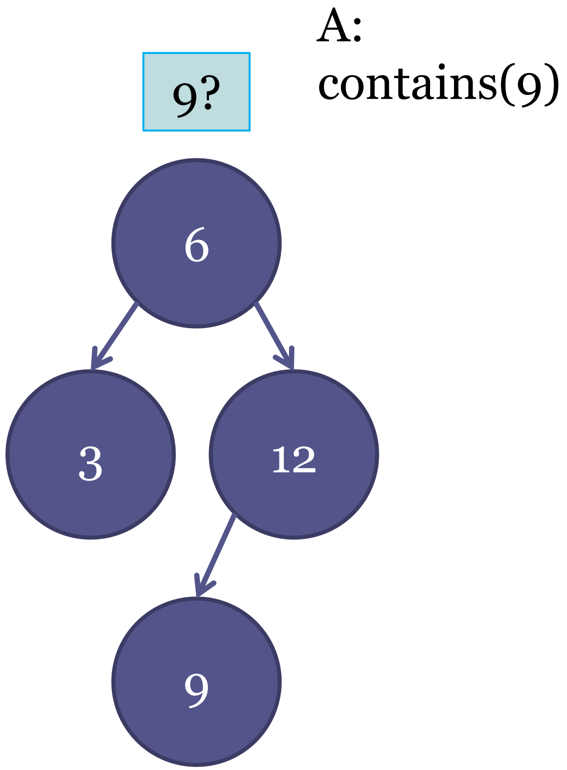

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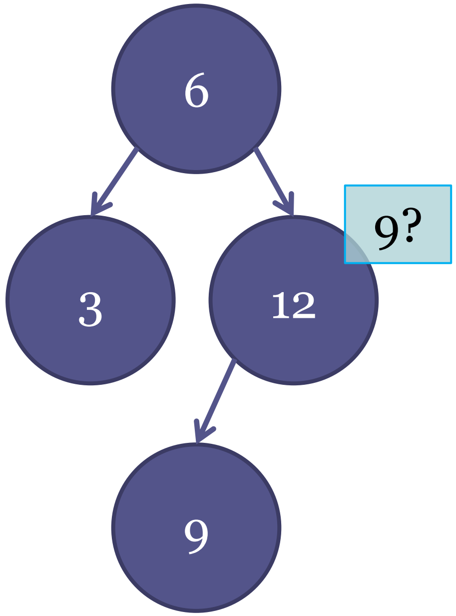

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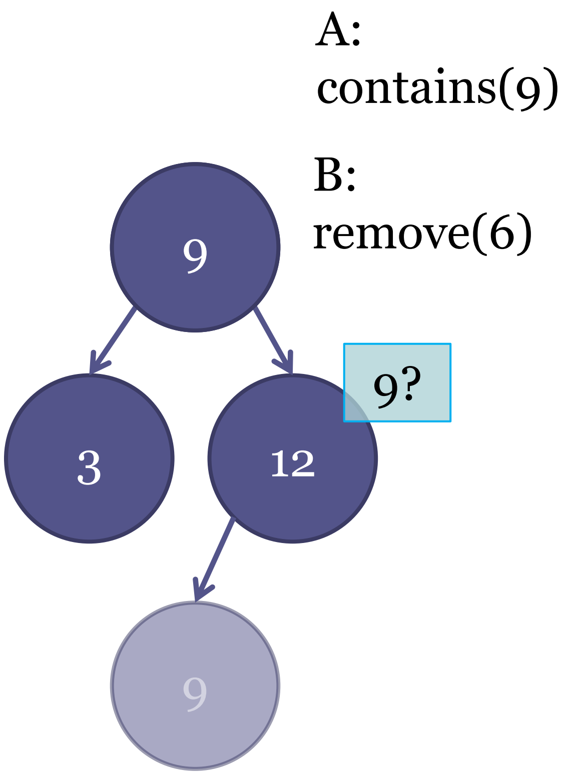

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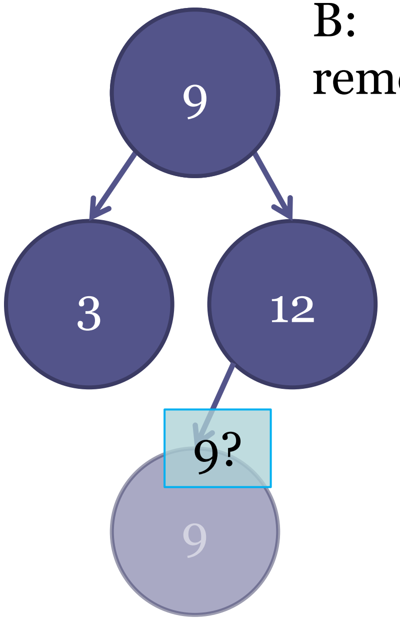
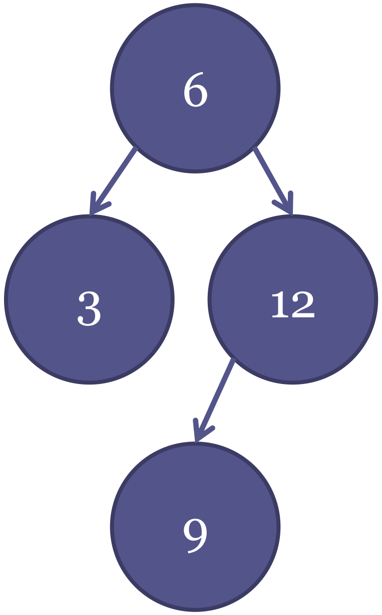


How do others cope with this challenge?

- These solutions leave removed nodes in the tree
- Is it possible to *physically* remove nodes?
- Trivial solution: use global lock



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- These solutions leave removed nodes in the tree
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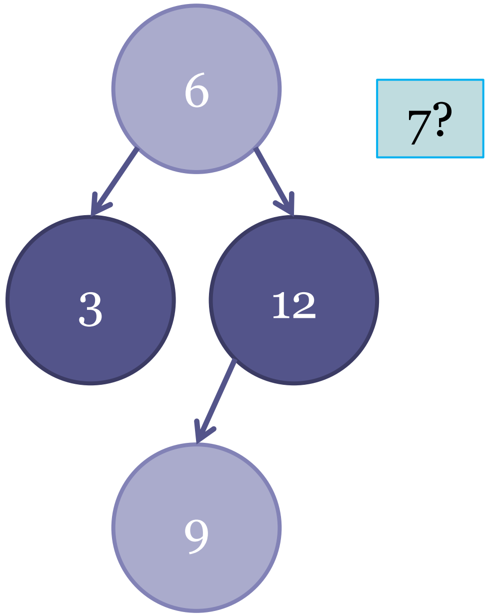
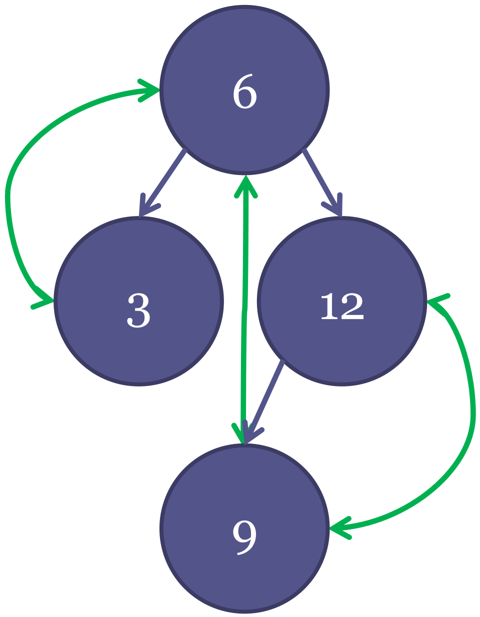
- Trivial solution: use global lock
- <u>Observation</u>: To determine whether k is in the tree it is enough to have p, s such that:
  - p, s belong to the tree
  - Any $w \in (p, s)$ is not in the tree



Our Approach

- Maintain the predecessor-successor relation
  - The set layout
- Consult this relation before making final decisions



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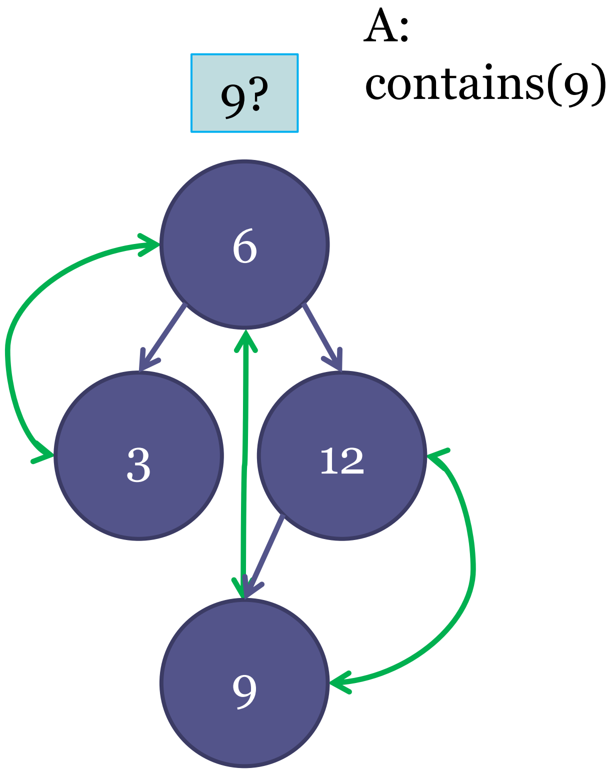

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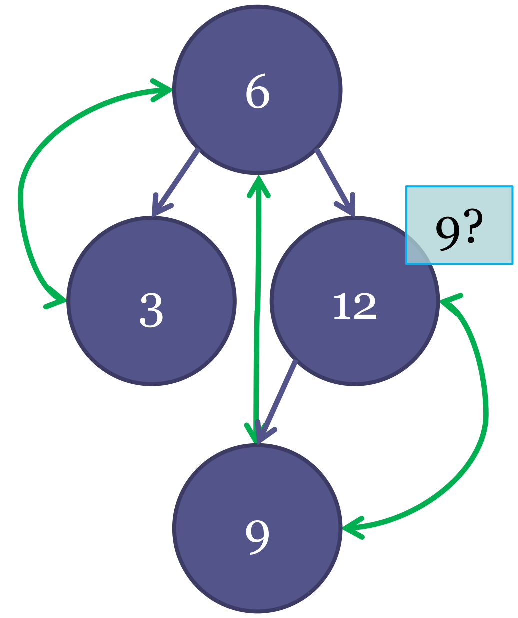

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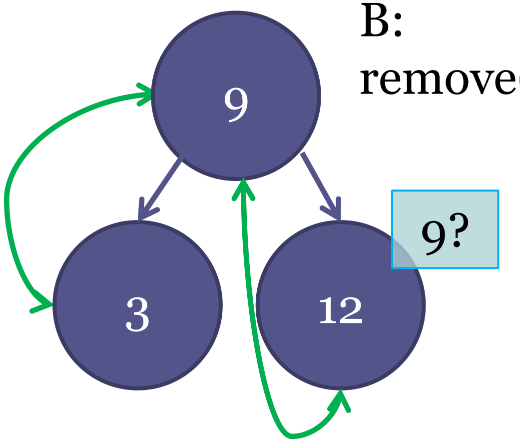

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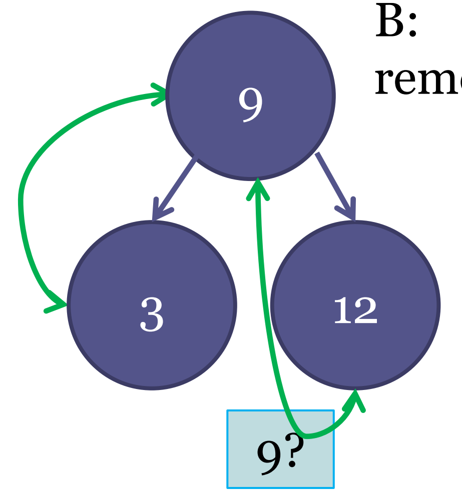

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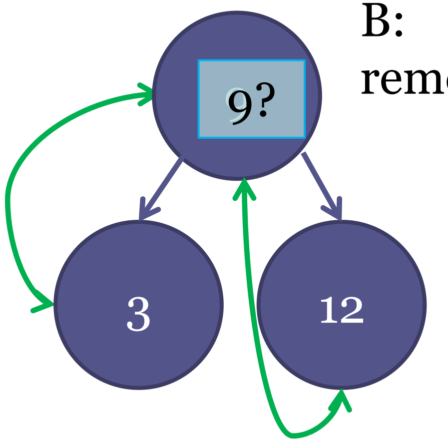
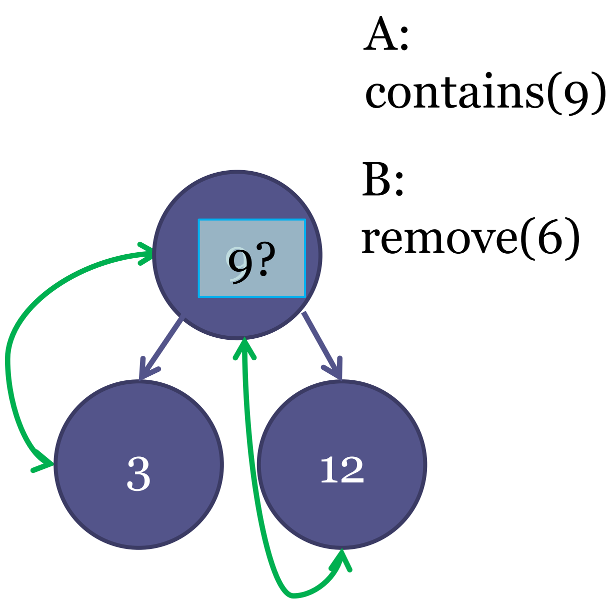


Our Approach

- Maintain the predecessor-successor relation
  - The set layout
- Consult this relation before making final decisions
- This relation allows us to lock the required nodes even if they are not adjacent
  - Enjoy the benefits of the global lock
  - While enabling more parallelism



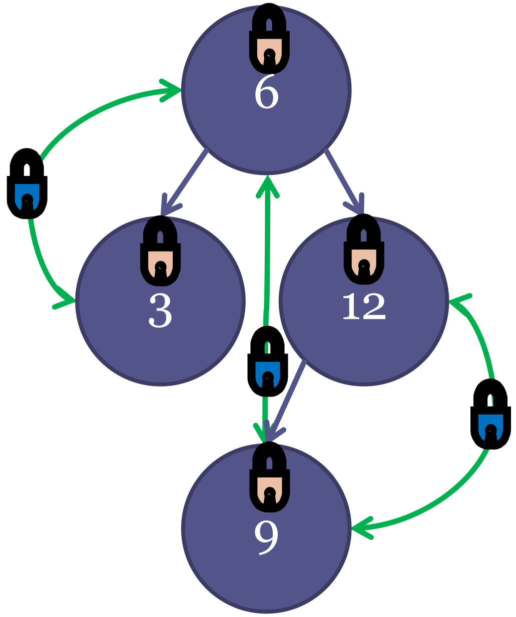
Contains(k)

- Traverse the tree using the tree pointers
- If k was found
  - Return true
- Otherwise, upon reaching to a leaf l, confirm:
  - $k \in (l'\text{s predecessor}, l)$ or $k \in (l, l'\text{s successor})$
  - and return false

- This operation does not acquire locks

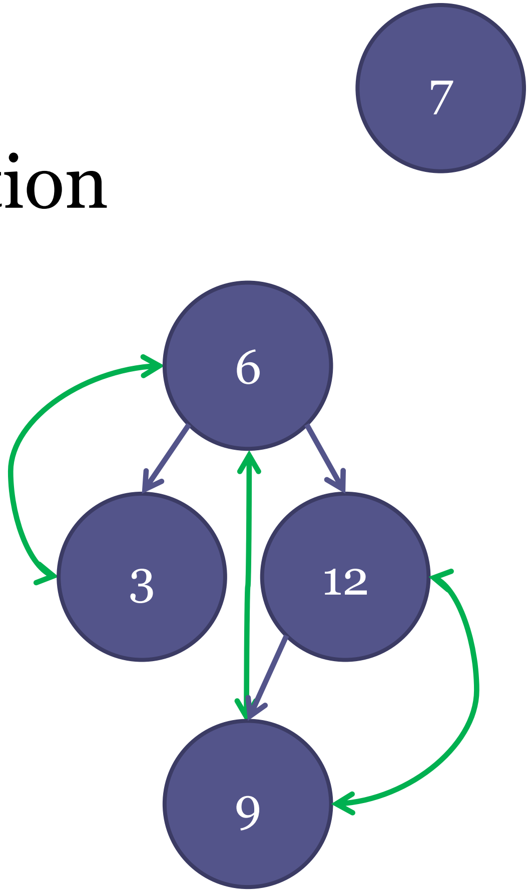
Update Operations

- The synchronization is based on locks
- Each update operation locks:
  - The relevant nodes in the tree
  - The relevant intervals



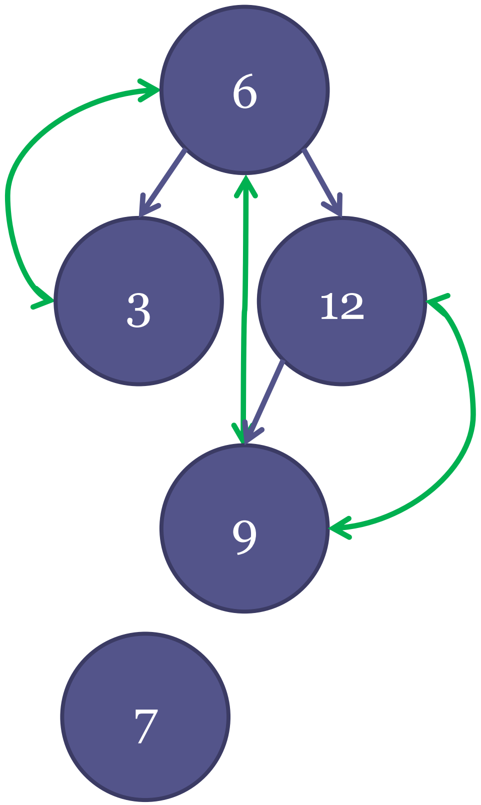
Insert(k)

- Traverse the tree to find the location



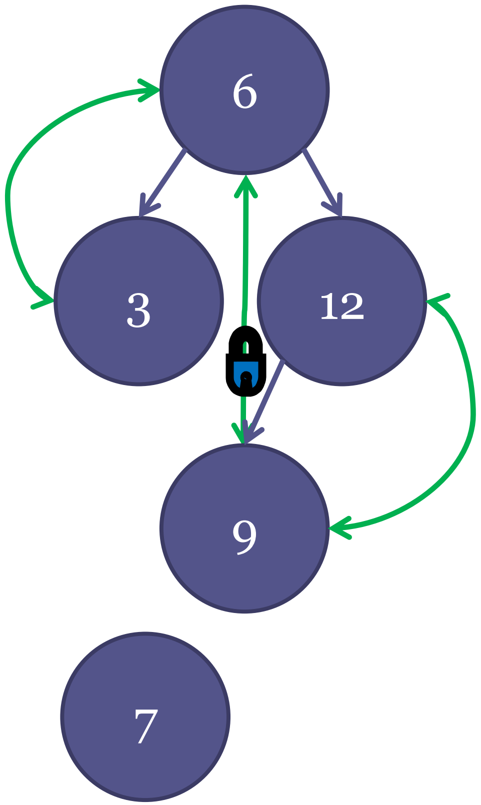
Insert(k)

- Traverse the tree to find the location
- Let l be the node found



Insert(k)

- Traverse the tree to find the location
- Let l be the node found
- If $k \leq l$: lock l's predecessor edge



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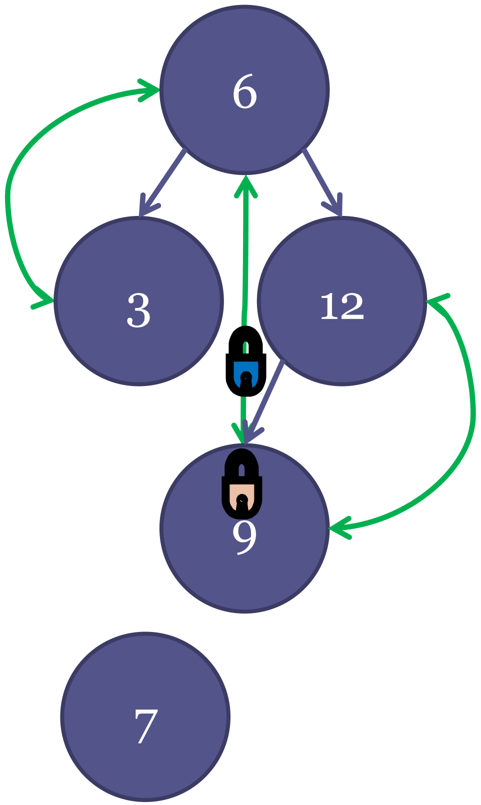
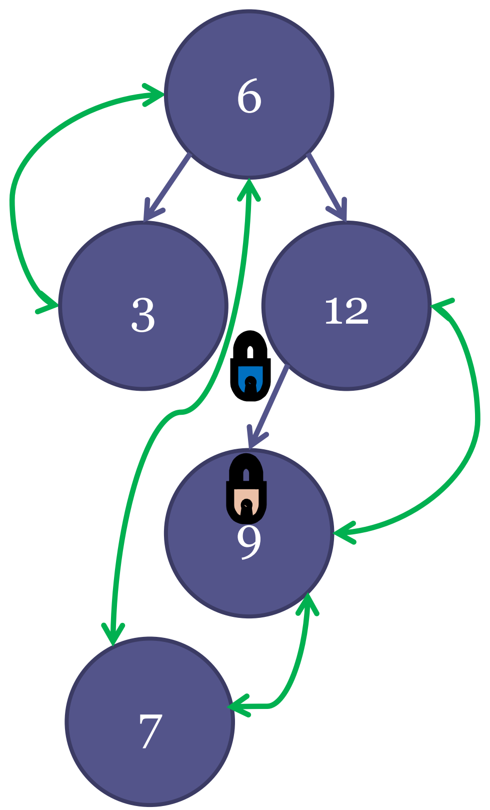


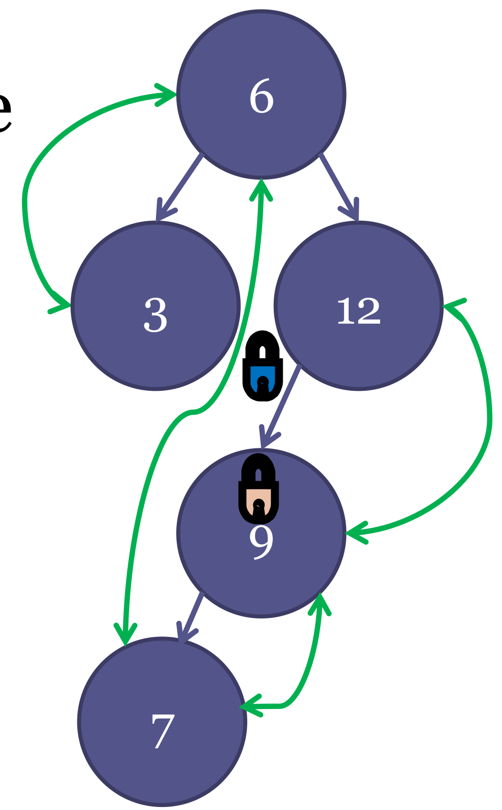
Insert(k)

- Traverse the tree to find the location
- Let l be the node found
- If $k \leq l$: lock l's predecessor edge
  - Lock l
  - Update predecessor-successor



Insert(k)

- Traverse the tree to find the location
- Let l be the node found
- If $k \leq l$: lock l's predecessor edge
  - Lock l
  - Update predecessor-successor
  - Add k



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- Let l be the node found
- If $k \leq l$: lock l's predecessor edge
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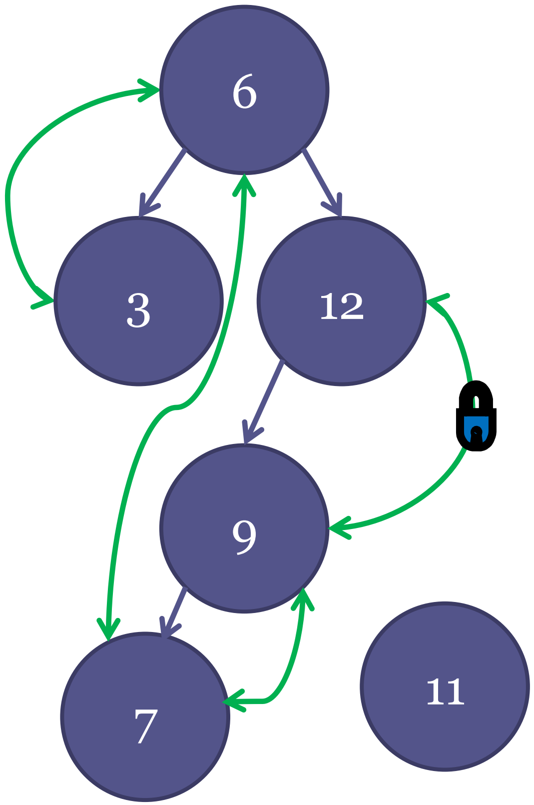
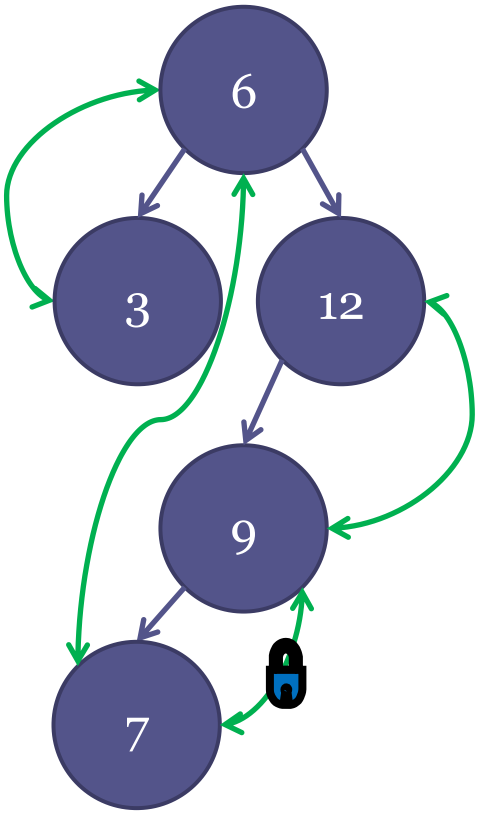
  - Update predecessor-successor
  - Add k
- Else: lock l's successor
  - Symmetric.



Remove(k)

- Traverse the tree to find k
- Let n be the node found
- Lock n's predecessor edge



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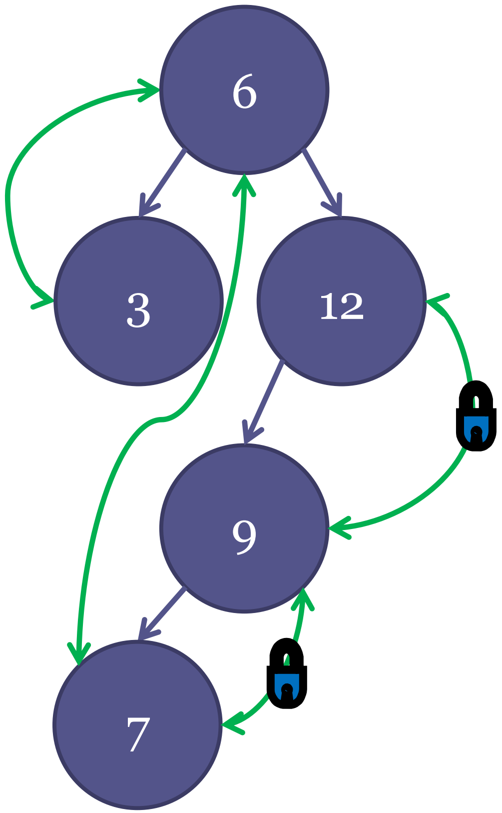
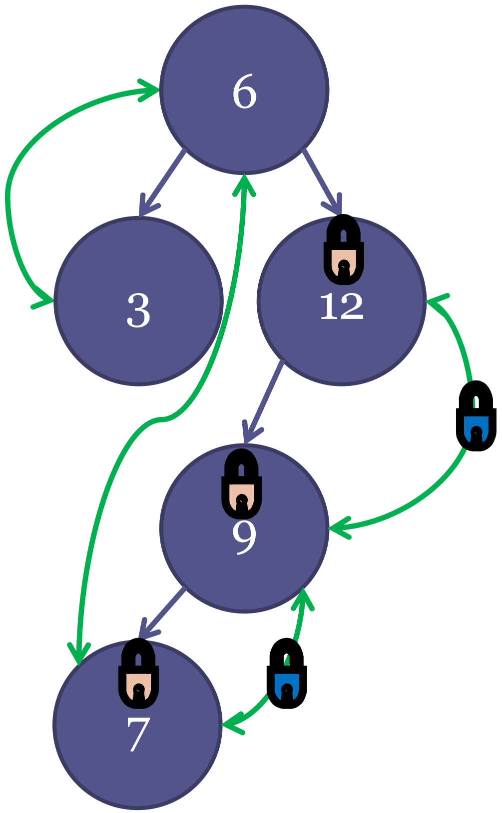


Remove(k)

- Traverse the tree to find *k*
- Let *n* be the node found
- Lock *n*'s predecessor edge
  - Lock *n*'s successor edge
  - Lock *n*, *n*'s children and parent



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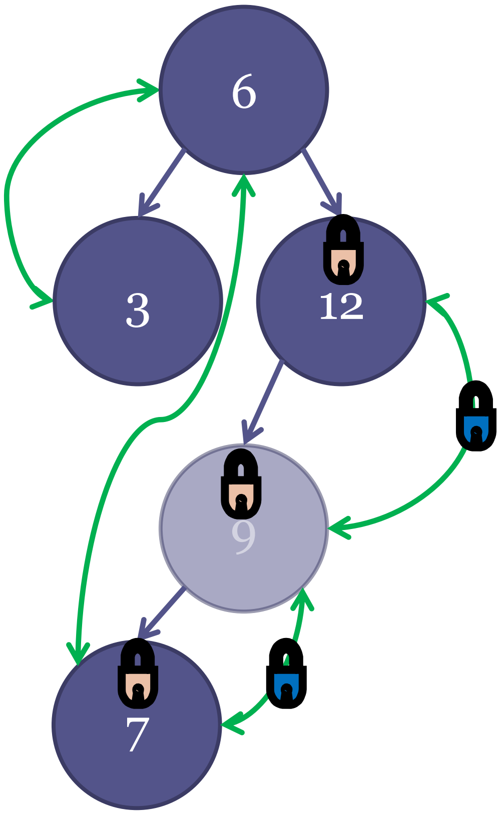
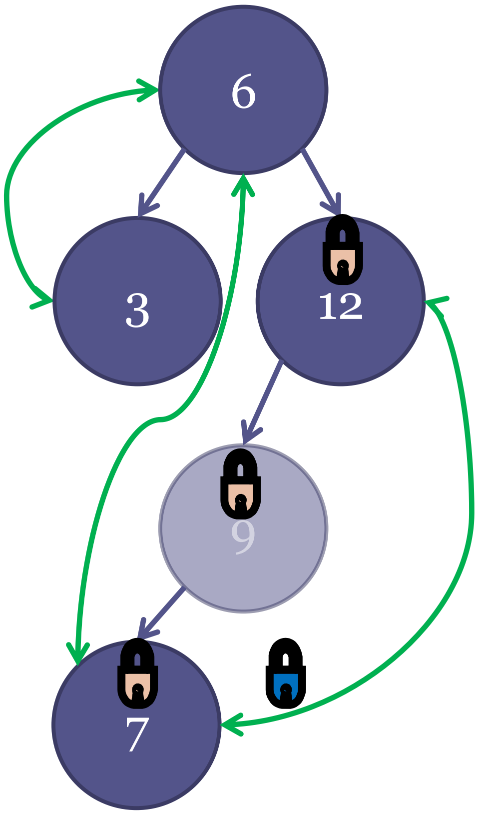
  - Lock n, n's children and parent
  - If n has at most 1 child:
    - Mark n as removed



Remove(k)

- Traverse the tree to find *k*
- Let *n* be the node found
- Lock *n*'s predecessor edge
  - Lock *n*'s successor edge
  - Lock *n*, *n*'s children and parent
  - If *n* has at most 1 child:
    - Mark *n* as removed
    - Update predecessor-successor



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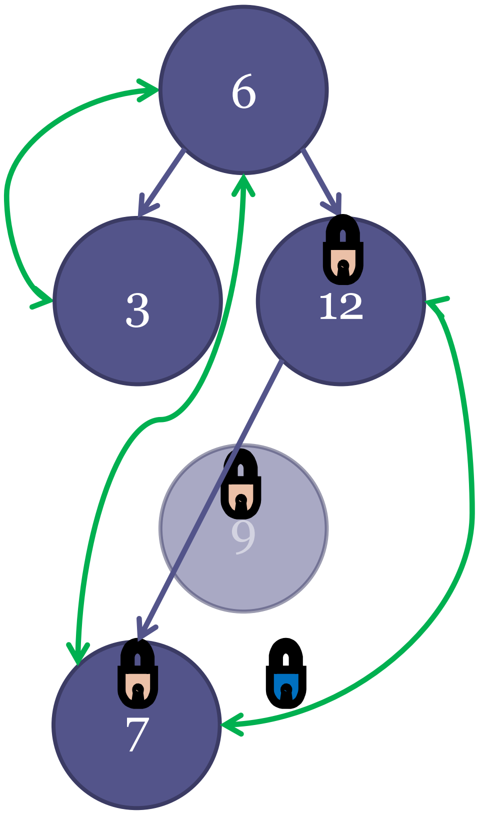
    - Connect n's parent and child



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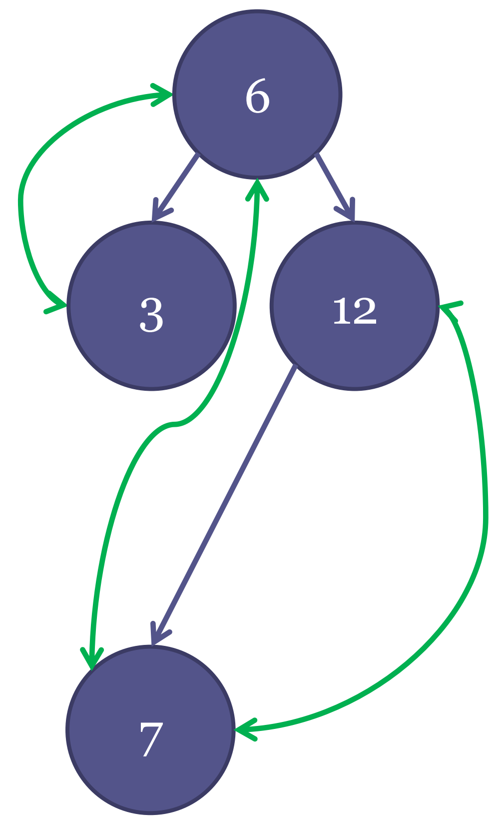
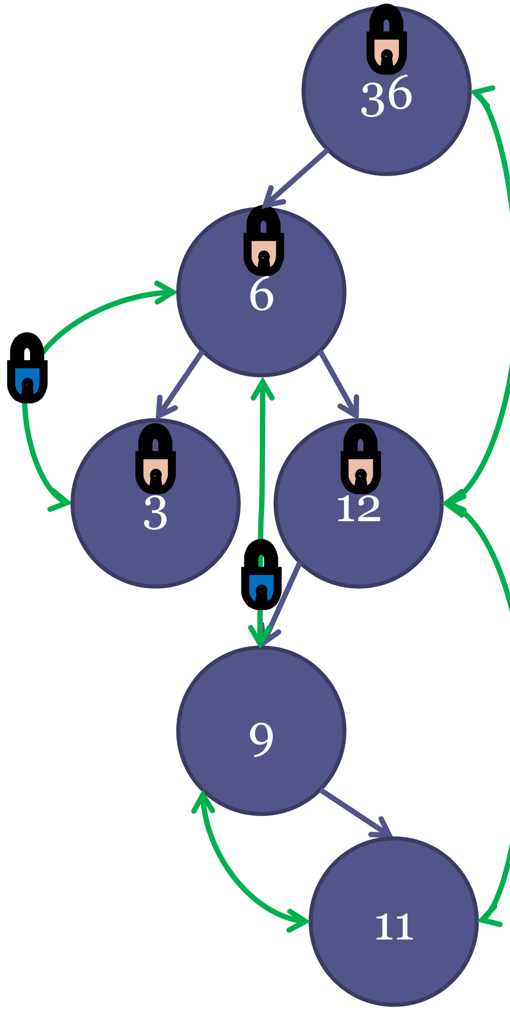


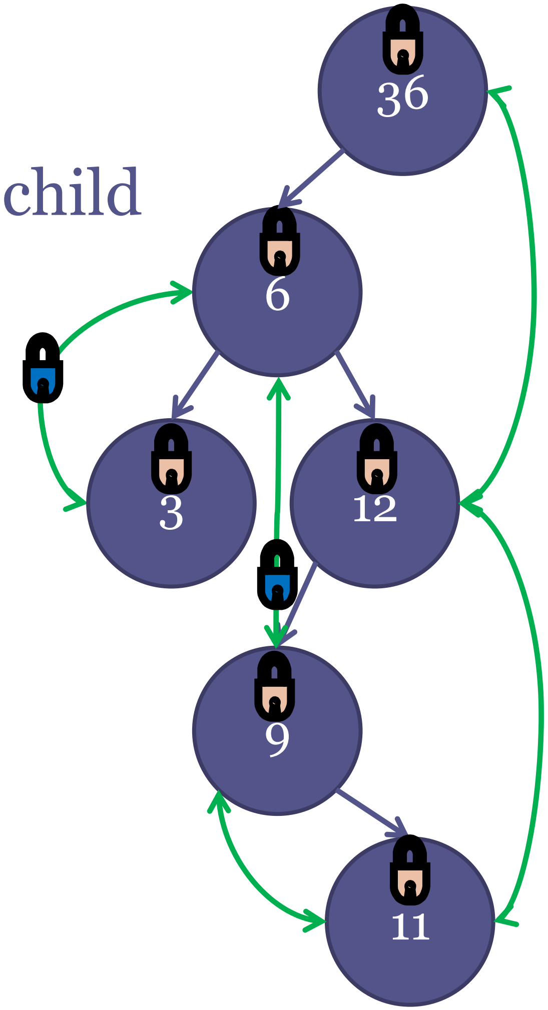
Remove(k)

- If n has 2 children:



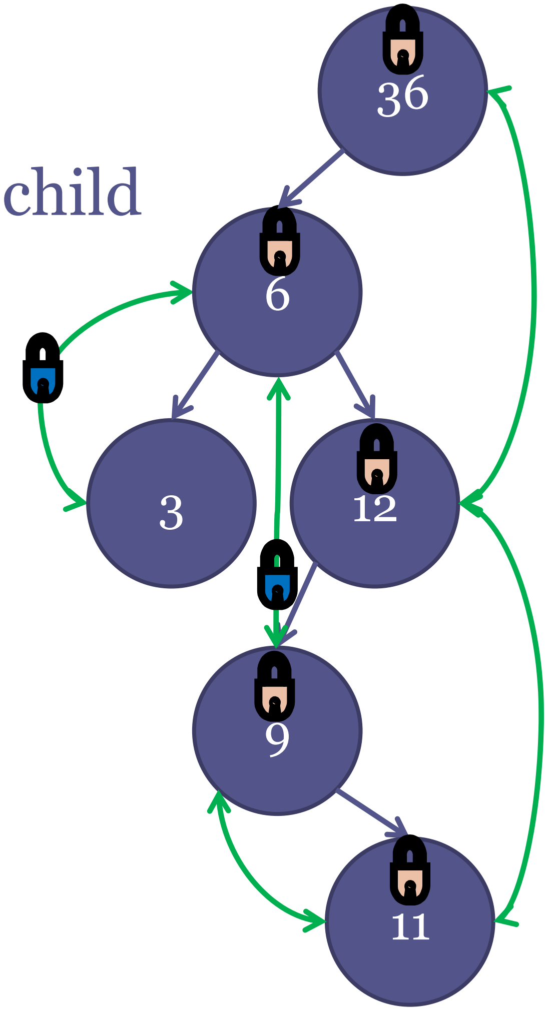
Remove(k)

- If n has 2 children:
  - Lock n's successor, its parent and child



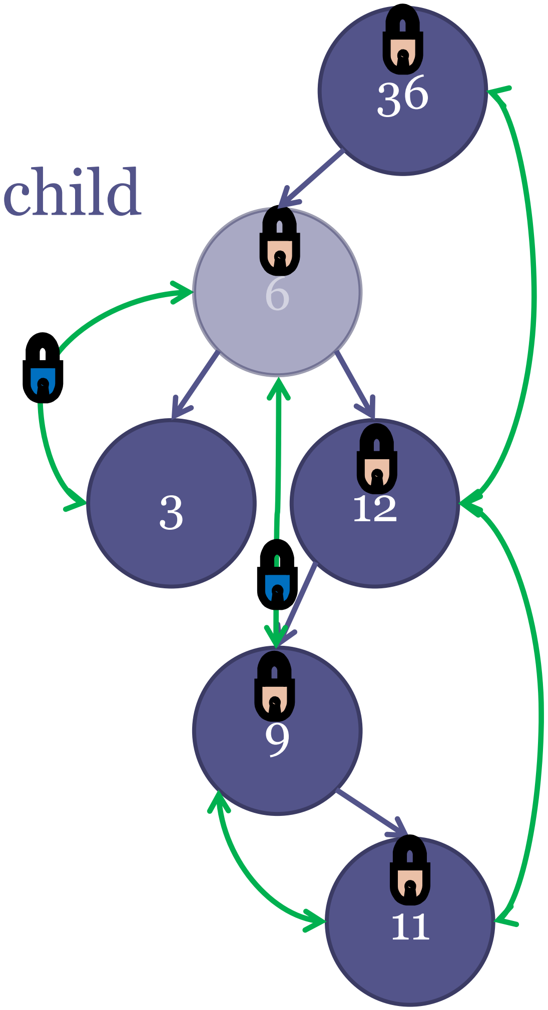
Remove(k)

- If n has 2 children:
  - Lock n's successor, its parent and child
  - Release n's children locks



Remove(k)

- If n has 2 children:
  - Lock n's successor, its parent and child
  - Release n's children locks
  - Mark n as removed



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- If n has 2 children:
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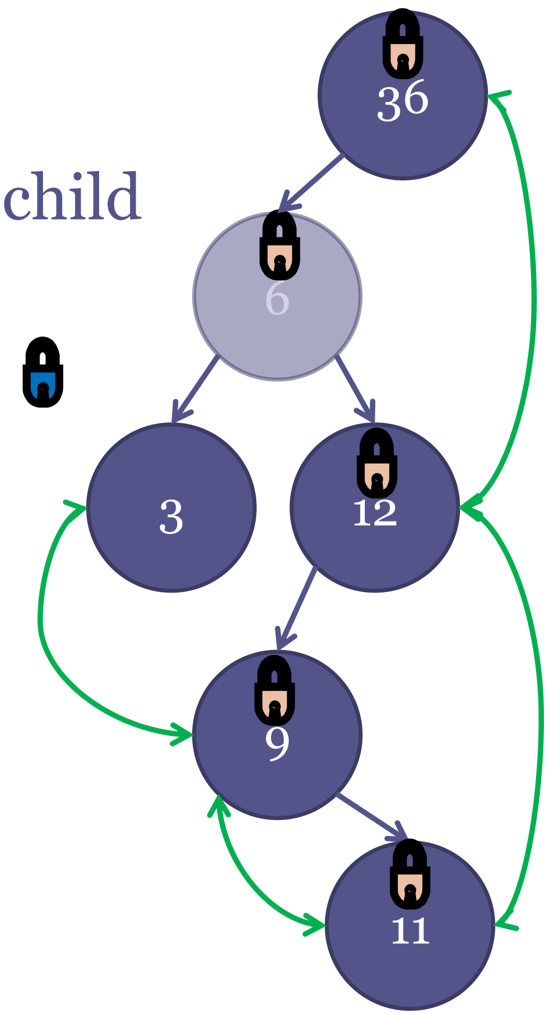
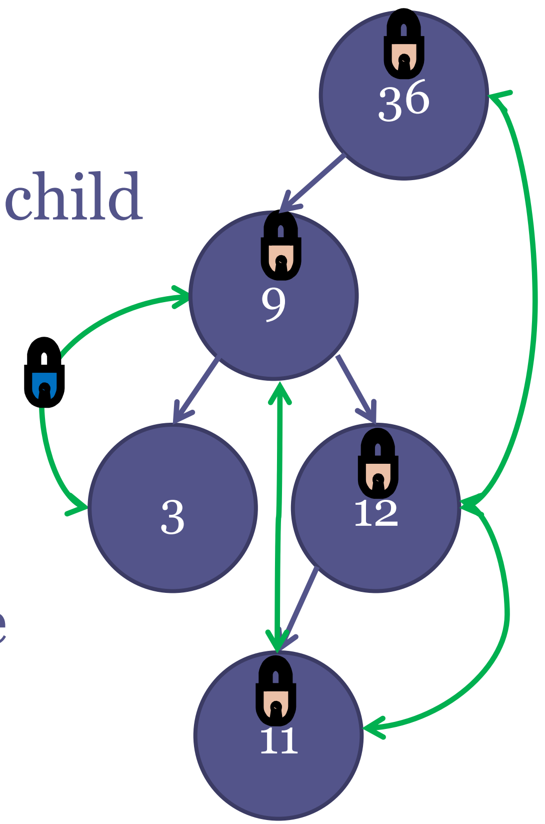
  - Release n's children locks
  - Mark n as removed
  - Update predecessor-successor



Remove(k)

- If n has 2 children:
  - Lock n's successor, its parent and child
  - Release n's children locks
  - Mark n as removed
  - Update predecessor-successor
  - Connect the successor's parent to the successor's child and relocate n's successor



Remove(k)

- If *n* has 2 children:
  - Lock *n*'s successor, its parent and child
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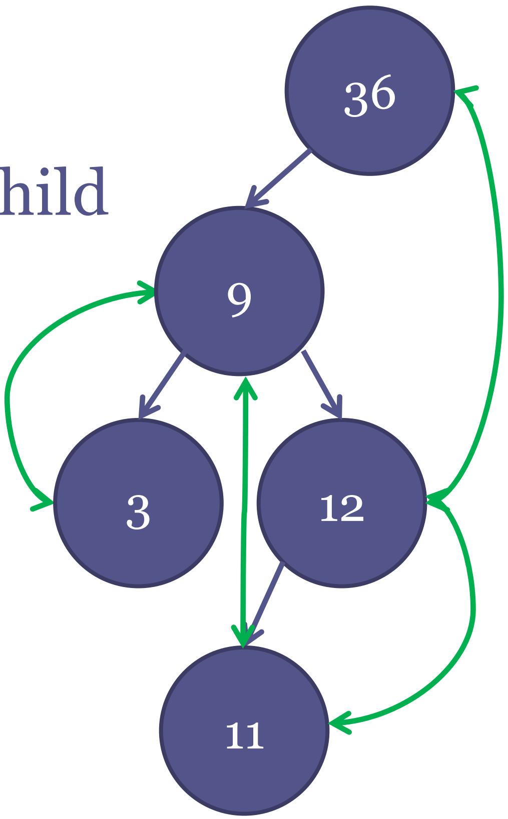
  - Update predecessor-successor
  - Connect the successor's parent to the successor's child and relocate *n*'s successor



Update Operations Scheme

- Traverse the tree to find k
- Lock interval: $[p, s]$
- Confirm that the interval is *appropriate*:
  - $k \in [p, s]$
  - p is not marked as removed
- Lock tree locks
- Update predecessor-successor relation
- Update tree layout
- Release all locks

Correctness

- The BST maintains two invariants
  - Set invariant
    - Protected by set-locks
  - BST invariants
    - Protected by tree-locks
- The intervals allow us to separate the proof into two proofs

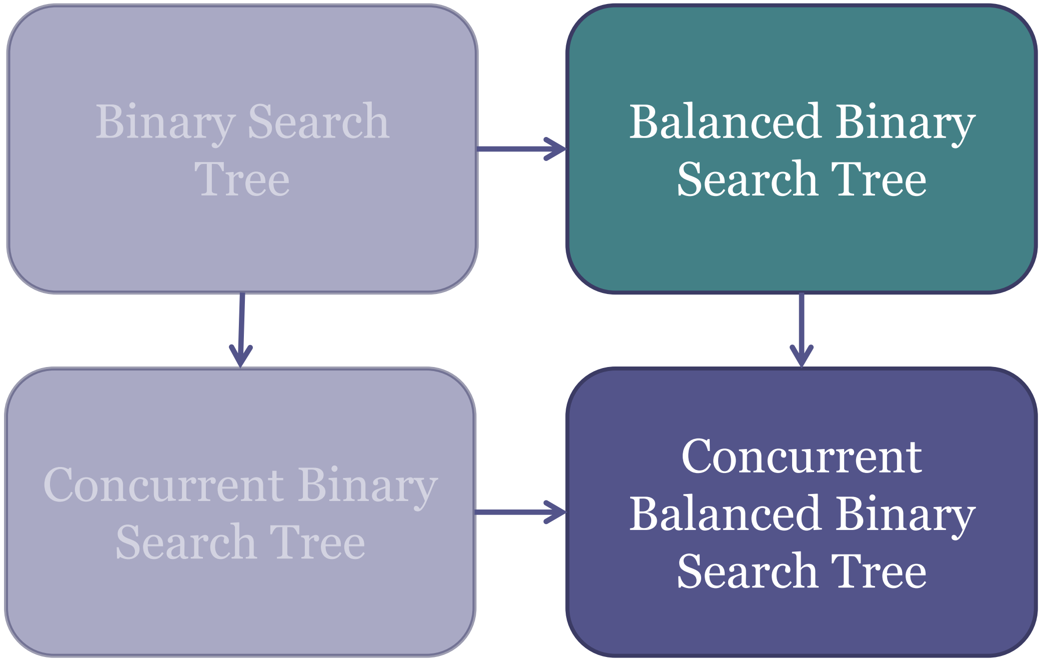
Correctness

- Set invariant
  - Each key appears at most once
- A new key, k, is added only after locking an interval $[p, s]$ such that $k \in (p, s)$
- k is not added if $k = p$ or $k = s$
- k cannot be added concurrently by another thread

Correctness

- BST invariants
  - For each node:
    - The keys in the left sub-tree are smaller
    - The keys in the right sub-tree are bigger
- The invariants may only be broken while updating the tree layout
- Any update operation locks all updated nodes
- Locks are released only after the BST invariants are held

Outline

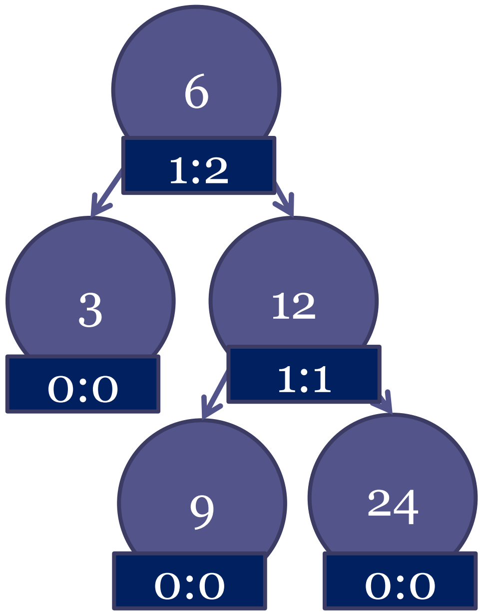

Binary Search Tree
Balanced Binary Search Tree
Concurrent Binary Search Tree
Concurrent Balanced Binary Search Tree

Balanced Binary Search Tree

- In BST, insert, remove and contains run in $O(\log n)$ in *average*.
- In balanced BST, these operations run in $O(\log n)$ in the *worst case*.
- There are several known implementations for balanced BSTs
  - We will focus on AVL trees

AVL Trees

- Each node maintains the invariant:
  - The heights of the left and right sub-trees differ by at most 1



AVL Trees

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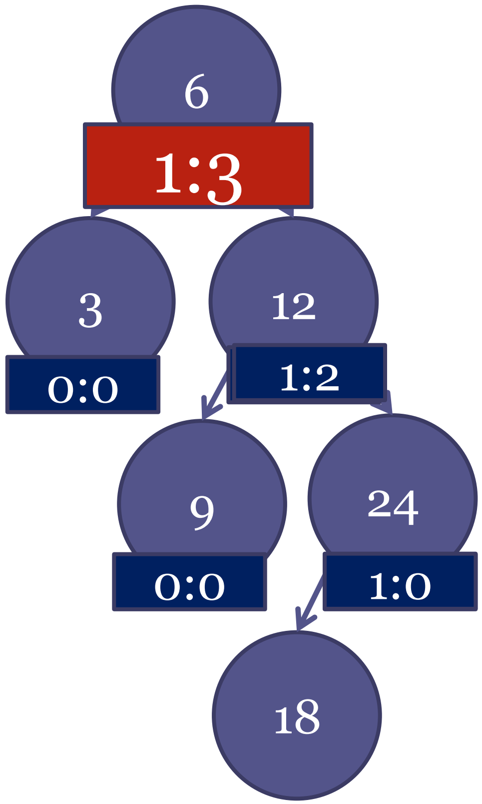
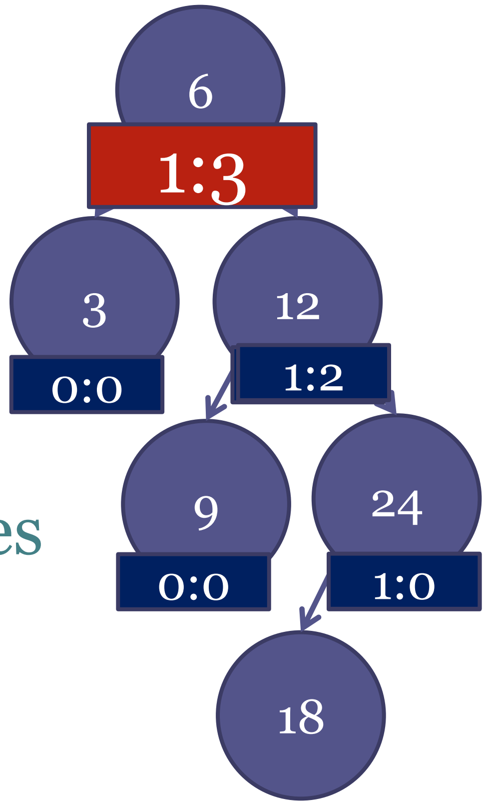
- Insertion and removal may break the invariant



AVL Trees

- Each node maintains the invariant:
  - The heights of the left and right sub-trees differ by at most 1
- Insertion and removal may break the invariant
  - Rotations are applied to fix it
  - Rotations operate on adjacent nodes



AVL Trees

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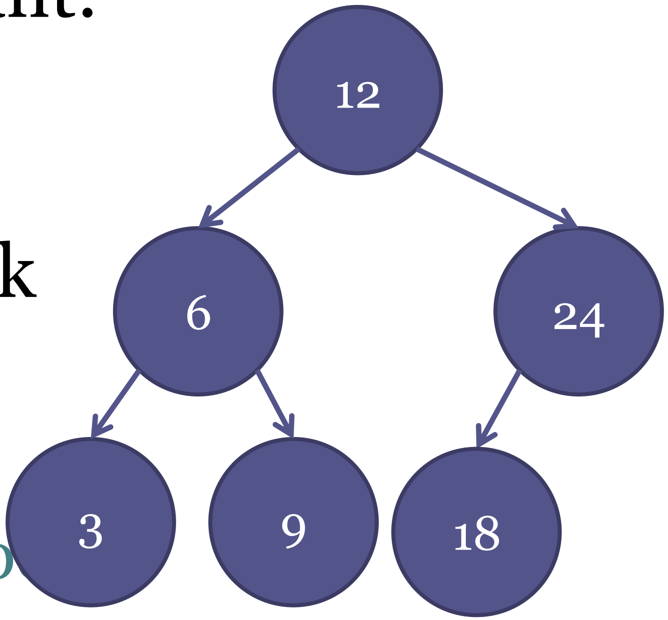
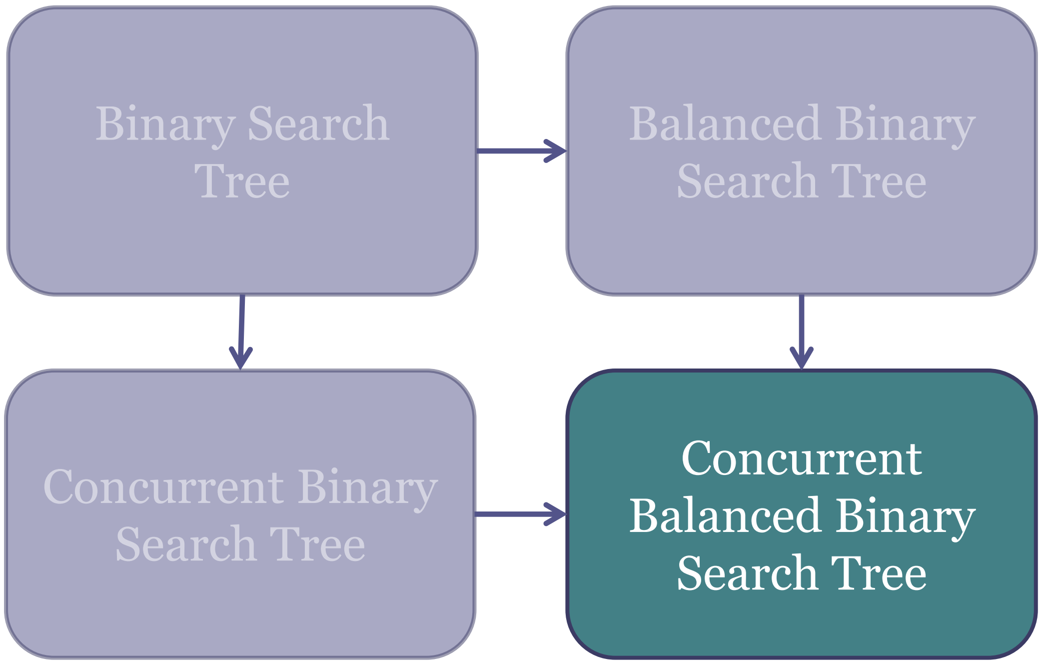


Outline


Binary Search Tree
Balanced Binary Search Tree
Concurrent Binary Search Tree
Concurrent Balanced Binary Search Tree

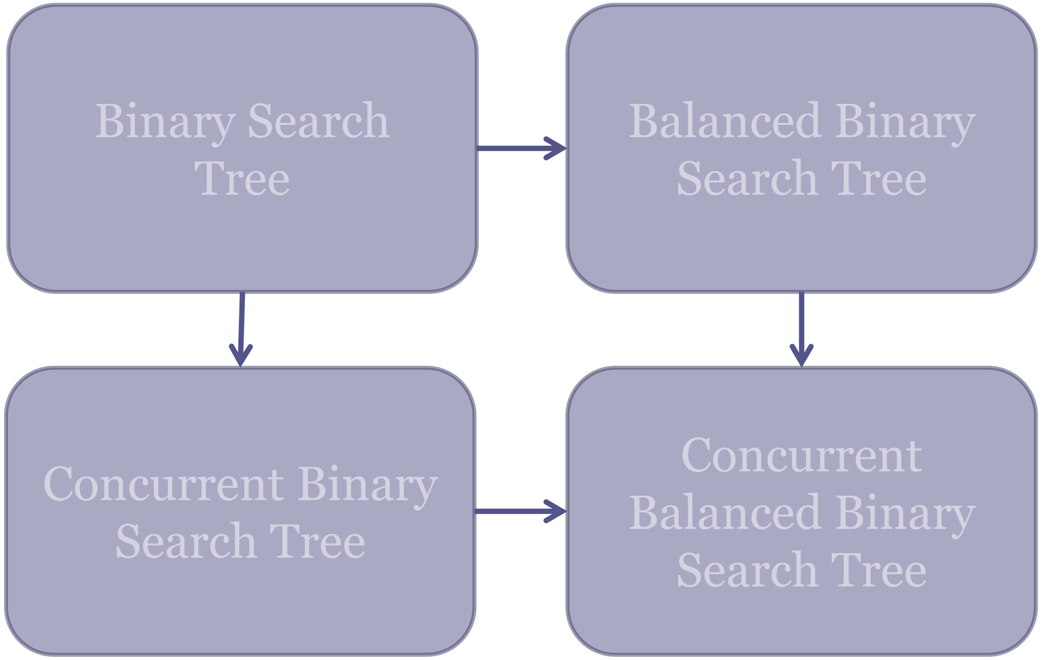
Balancing Our Tree

- After insertion or removal the tree is traversed bottom-up beginning from the point where an update has occurred
- If violation is detected, rotations are applied
  - Only tree layout locks need to be acquired

Balancing Our Tree

- Rotations may lead to temporary disappearance of nodes from the tree layout
- However, the set-layout is unaffected by these rotations
- Since we consult the set-layout before making final decisions, this cannot lead to wrong decisions

Overview



Binary Search Tree
Balanced Binary Search Tree
Concurrent Binary Search Tree
Concurrent Balanced Binary Search Tree

Evaluation

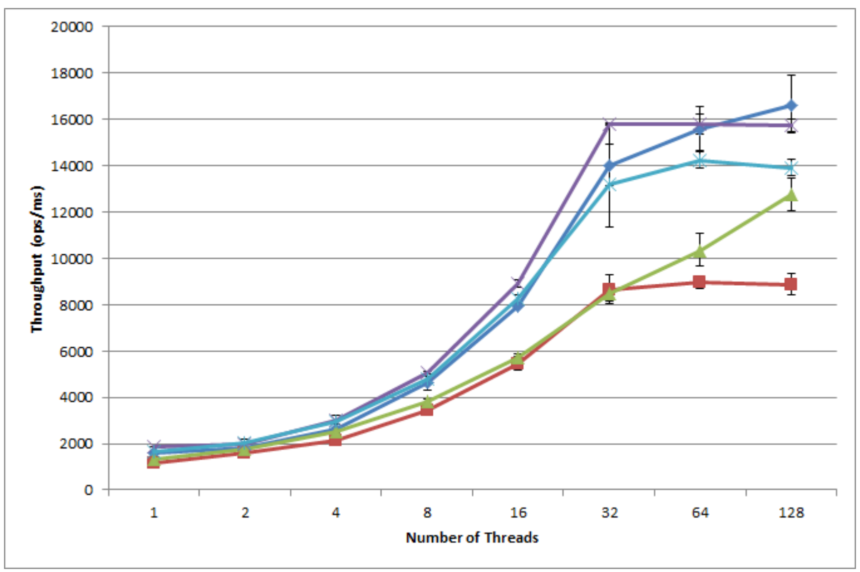
- We compared our tree to state-of-the-art implementations
- Experiments ran on a machine with 32 cores

Evaluation

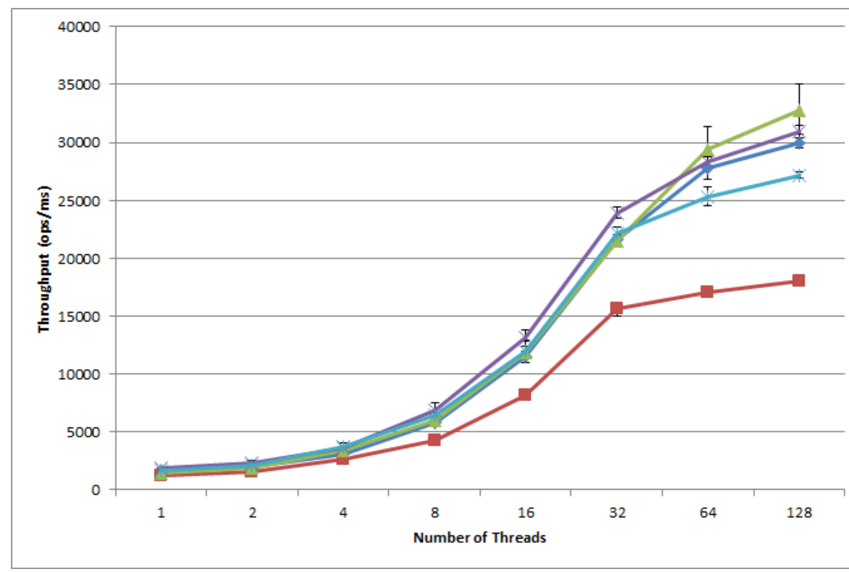
- 90% contains, 9% insert, 1% remove

Bronson's Tree · Skip List · Contention Friendly Tree · Our AVL · Our Tree

200,000 keys



2,000,000 keys



Summary

- We presented a practical concurrent balanced BST
- Our main insight is that maintaining explicitly the set layout results in a simpler algorithm for the concurrent balanced BST

Thank you!

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