TITLE: From simple combinatorial statements with difficult mathematical proofs to hard instances of SAT

ABSTRACT: In this presentation I will present work in progress on the proof complexity of a class of unsatisfiable propositional formulas denoted by $Kneser_{n,k}$. These formulas (in n variables) are parameterized by a constant integer $k\geq 1$ and generalize the pigeonhole principle (obtained when $k=1$). They encode a combinatorial principle known as Kneser-Lovasz theorem (conjectured by Kneser, later proved by Lovasz using techniques from algebraic topology) Our results are as follows:

- we prove exponential lower bounds for the resolution complexity of any family $Kneser_{n,k}$. A standard connection yields lower bounds for the complexity of DPLL algorithms as well. Such results hold in fact for a stronger version of the Kneser-Lovasz theorem proved by Schrijver.

- similarly, an exponential lower bounds holds for the complexity of so-called bounded-depth Frege proofs.

On the other hand

- Formulas $Kneser_{n,k}$ with k=2 have polynomial size Frege proofs.
- Formulas $Kneser_{n,k}$ with k=3 have polynomial size extended Frege proofs.

- for $k\geq 4$ for which no purely combinatorial proof of the Kneser Lovasz conjecture is known we show that the corresponding formula have polynomial size proofs in a proof system at the second level of Krajicek's hierarchy of implicit extended Frege proofs.

I will also discuss the prospects of (and practical issues related to) using such formulas as benchmarks for SAT solvers, as well as related open issues.

Joint work with Adrian Craciun (West University of Timisoara)