Verification of Mixed Discrete-Continuous Systems

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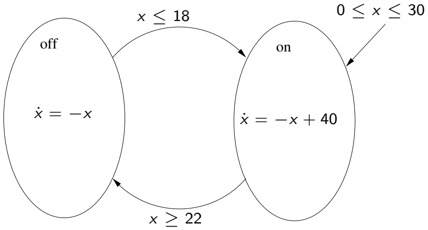
Hybrid (Dynamical) Systems

Example: Thermostat



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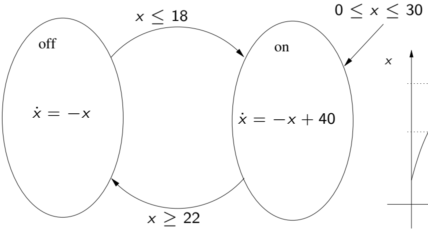

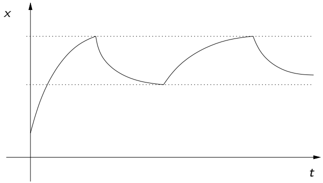

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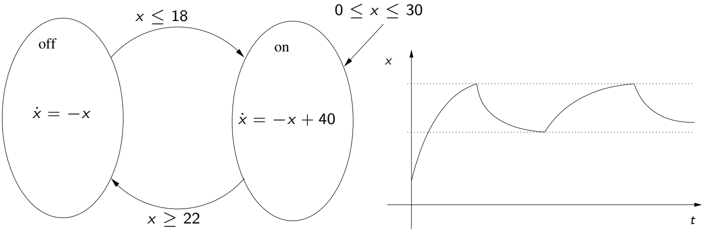

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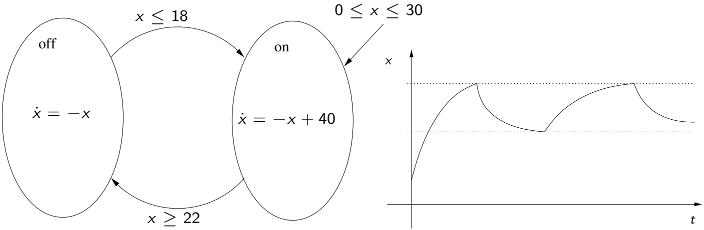

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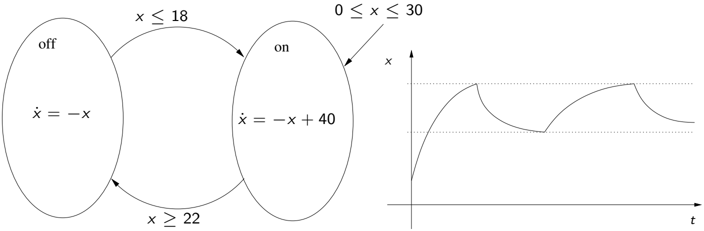

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But: software more complex than finitely many modes

Talk Outline

- Our method for safety verification of hybrid systems

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- Speculation: More interesting discrete behavior?

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Tool for safety verification of hybrid systems (hsolver.sourceforge.net,[Ratschan and She, 2007])

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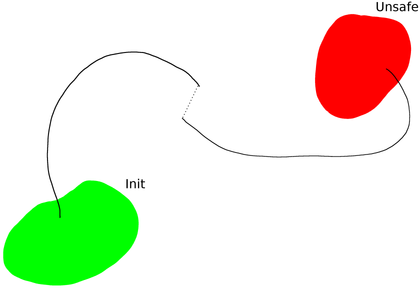

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Output:

- If terminates (printing "safe") then there is no *error trajectory* (i.e., trajectory from initial to unsafe states).
- Might run forever.

Characteristics of HSolver

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Weaknesses:

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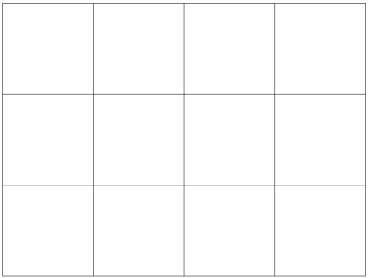
- Only restricted method for finding error trajectories [Ratschan and Smaus, 2009]
- Current implementation: does not scale wrt. number of discrete modes

Abstractions in HSolver

Box grid [Kuipers, 1995]

2-dimensional, 1 mode:



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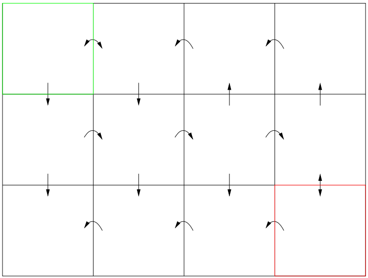

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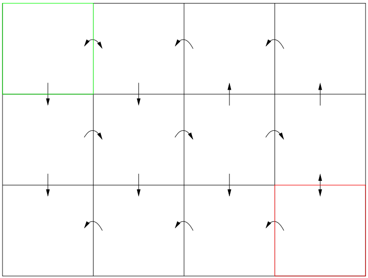

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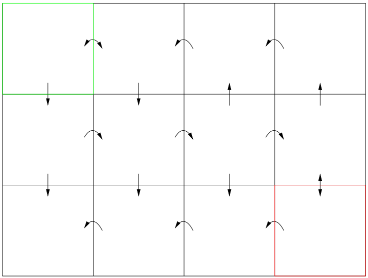

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refinement: split a box into two
increases abstraction size: only use as last resort!

Abstraction Pruning

Reflect more information in abstraction,
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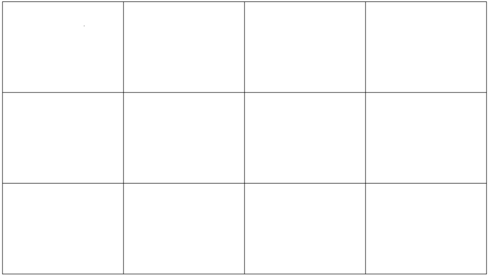

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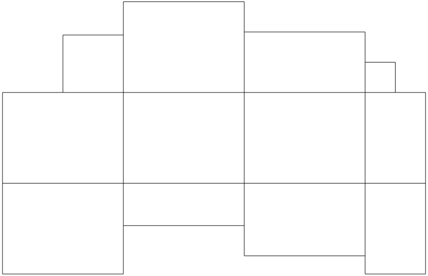

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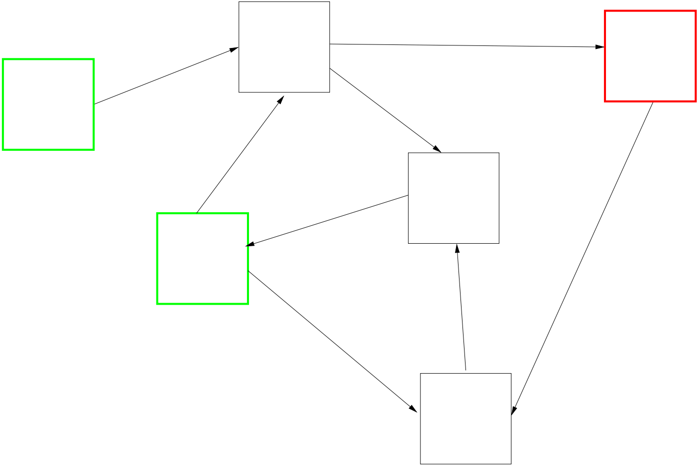

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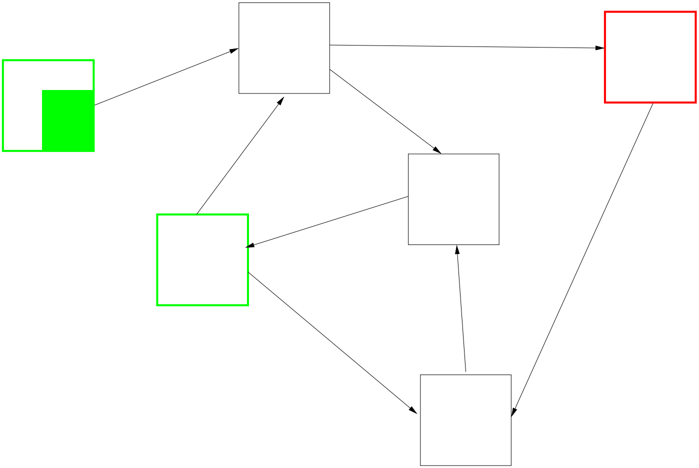

For each box marked as initial:
  over-approximate set of states reachable from an initial state

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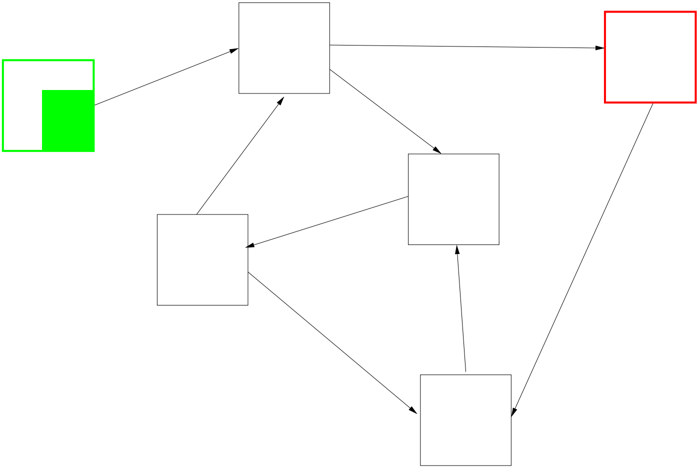

If empty set, remove initiality mark

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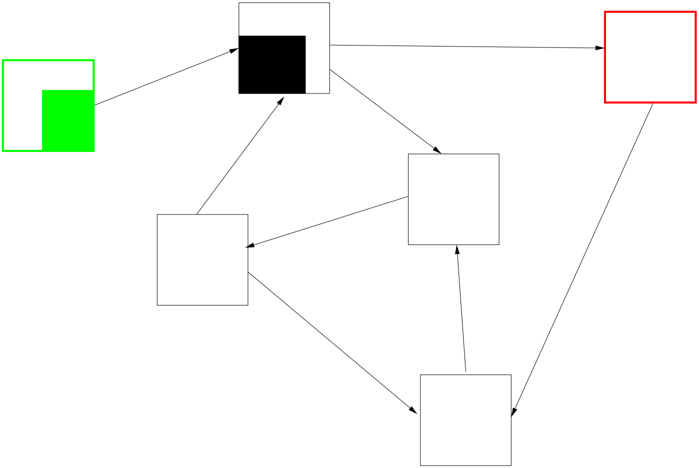

while some new states reachable through a transition,
add them

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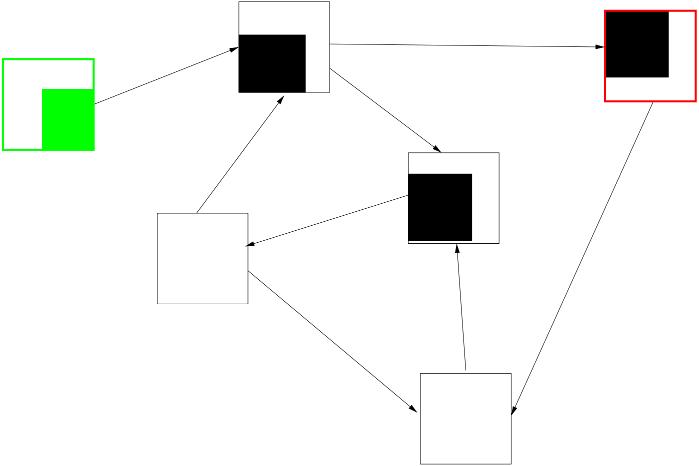

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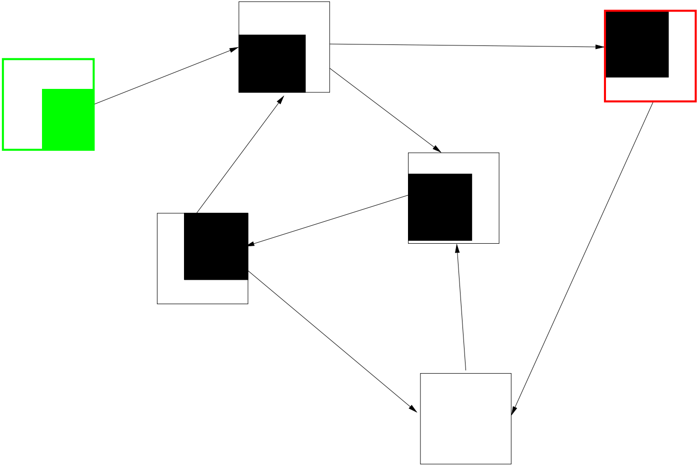

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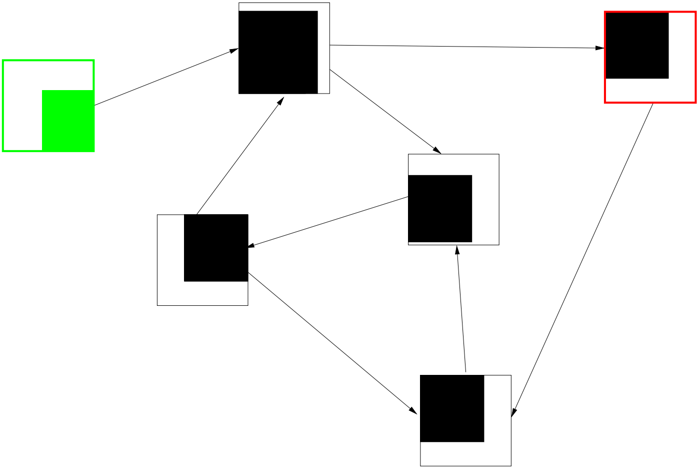

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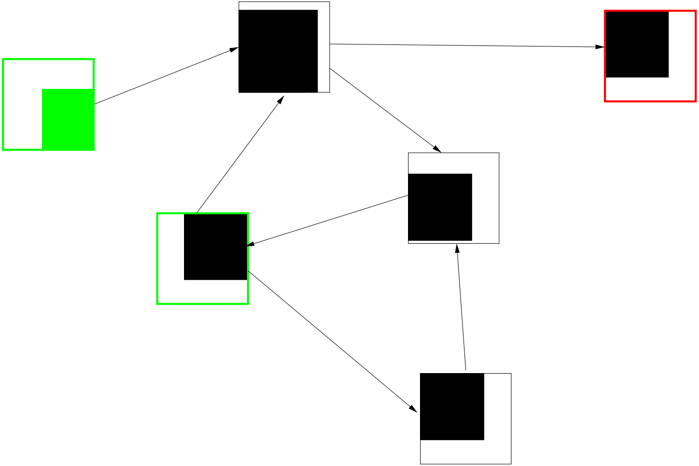

remove unconfirmed transitions

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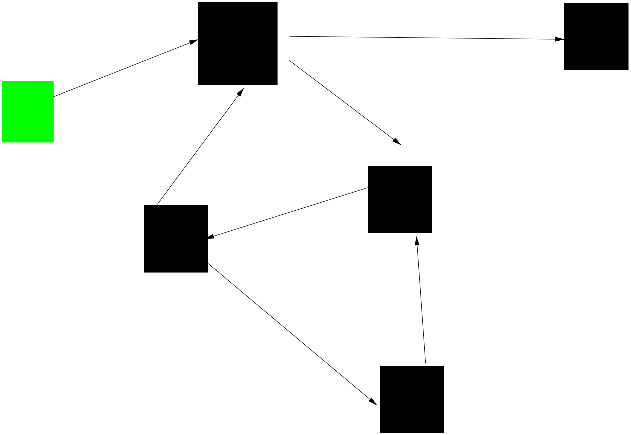

Replace boxes by new ones

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Error trajectory: starts in initial states, leads to unsafe states

Hence: apply pruning algorithm also backward in time

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Our implementation:

- Interval constraint propagation [Benhamou and Granvilliers, 2006] +
- algebraization of ODEs [Hickey, 2000, Ratschan and She, 2007]

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How to match discrete to continuous time?

For each program statement interval bound on duration?
(may be $[0, 0]$)

Previous Work

HybridFluctuat (Bouissou, Goubault, Putot et. al.)

Similar model for interaction software - environment.

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Similar model for interaction software - environment.

Verification by symbolic co-simulation

Interval based ODE integrator (i.e., good for deterministic systems, only limited non-determism)

Bounded time

Does not exploit property at hand

Thought Experiment: Software Analogon of Verif. Alg.

Naive translation from hybrid to software.

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- $\sqsubseteq$ s.t. $a_1 \sqsubseteq a_2$ implies $a_1 \subseteq a_2$

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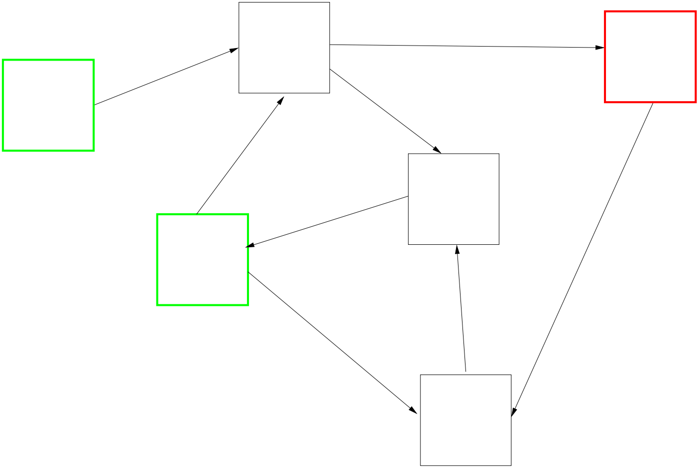

Boxes now represent elements of a chosen class $\mathcal{A}$ of subsets of Ω

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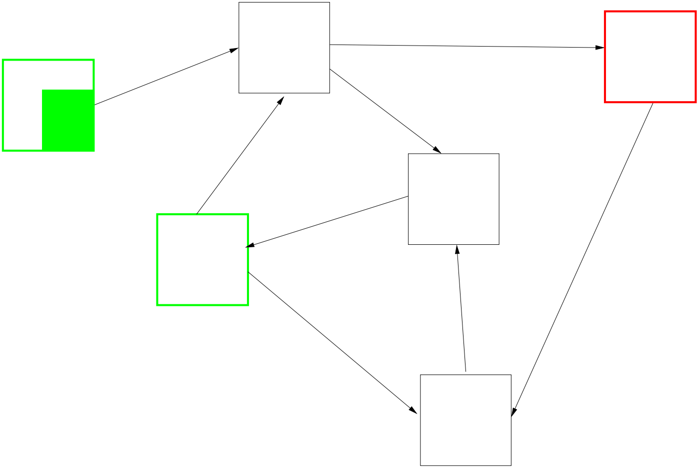

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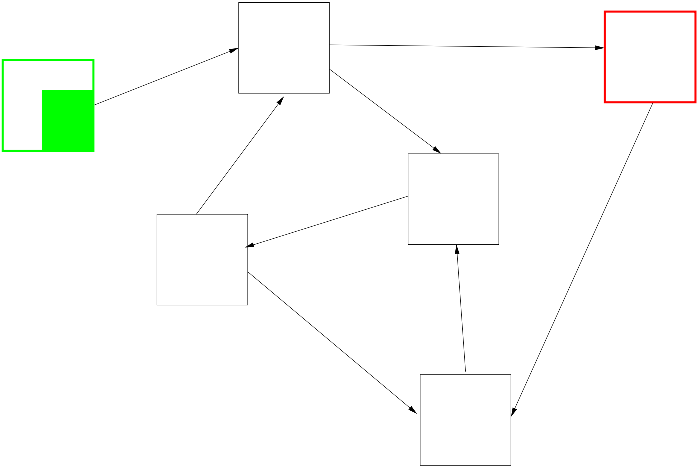

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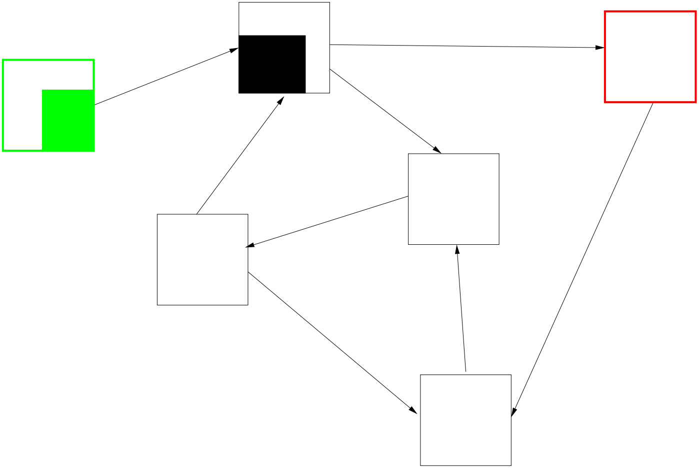

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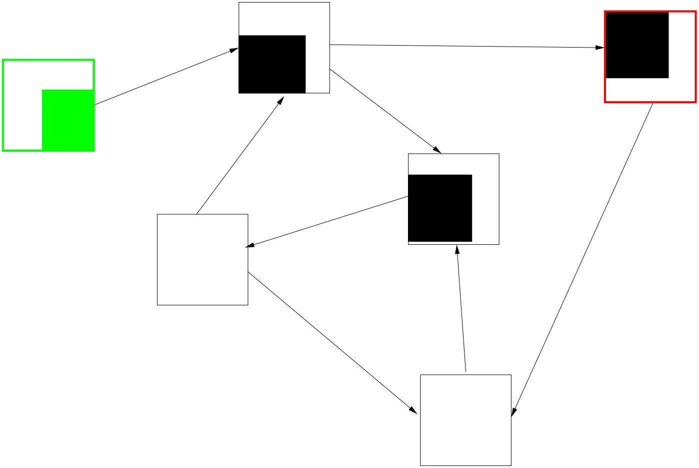

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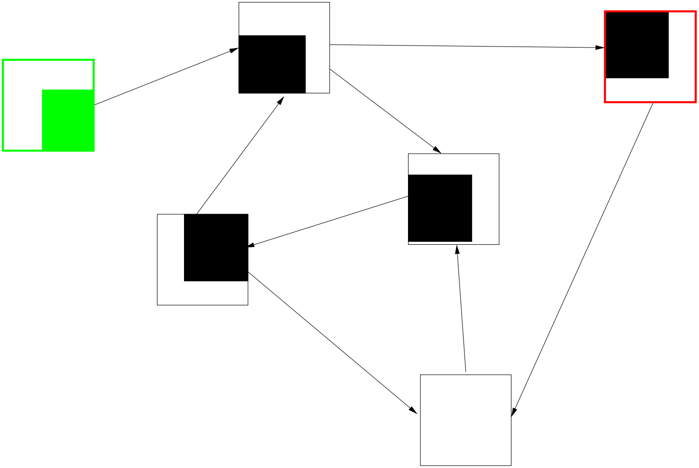

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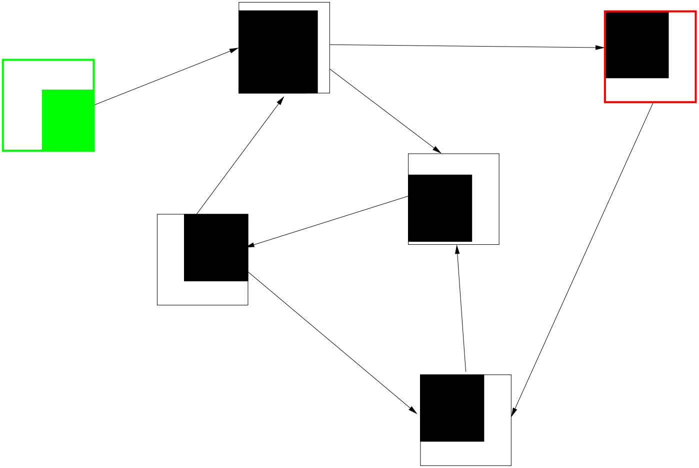

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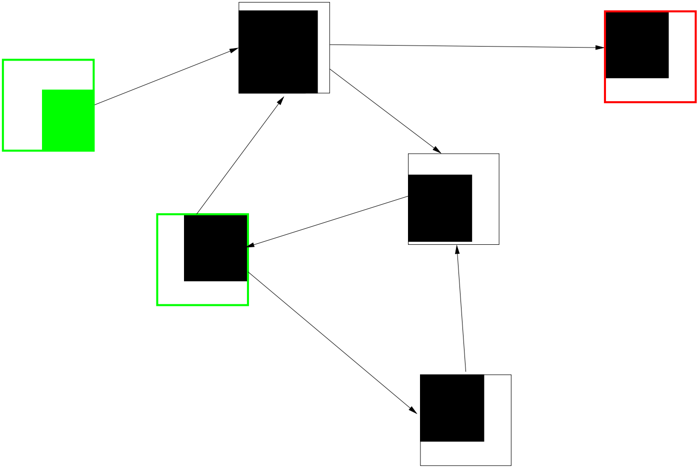

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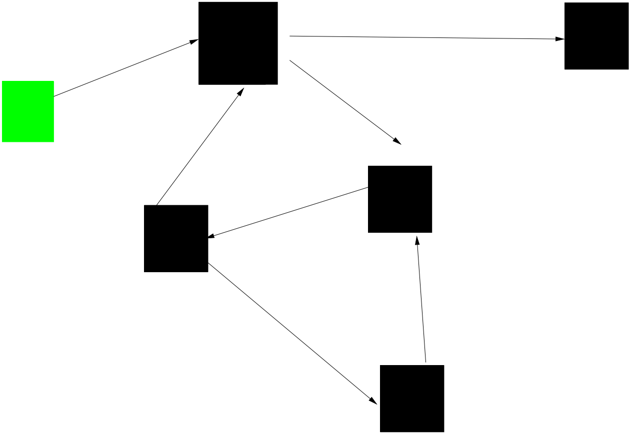

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- Which class of abstract states $\mathcal{A}$?

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- Is all of this anyway completely nonsense?

Literature I

F. Benhamou and L. Granvilliers. Continuous and interval constraints. In F. Rossi, P. van Beek, and T. Walsh, editors, *Handbook of Constraint Programming*, chapter 16, pages 571–603. Elsevier, Amsterdam, 2006.

Olivier Bouissou, Eric Goubault, Sylvie Putot, Karim Tekkal, and Franck Védrine. HybridFluctuat: A static analyzer of numerical programs within a continuous environment. In *Proceedings of Computer Aided Verification CAV'09*, volume 5649 of *LNCS*, pages 620–626. Springer, 2009.

Tomáš Dzetkulič and Stefan Ratschan. How to capture hybrid systems evolution into slices of parallel hyperplanes. In *ADHS'09: 3rd IFAC Conference on Analysis and Design of Hybrid Systems*, pages 274–279, 2009.

Literature II

Timothy J. Hickey. Analytic constraint solving and interval arithmetic. In *Proc. of the 27th ACM SIGACT-SIGPLAN Symp. on Principles of Progr. Lang.*, pages 338–351. ACM Press, 2000.

B. J. Kuipers. Qualitative simulation. *Artificial Intelligence*, 29: 290–338, 1995.

Stefan Ratschan and Zhikun She. Safety verification of hybrid systems by constraint propagation based abstraction refinement. *ACM Transactions in Embedded Computing Systems*, 6(1), 2007.

Stefan Ratschan and Jan-Georg Smaus. Finding errors of hybrid systems by optimising an abstraction-based quality estimate. In Catherine Dubois, editor, *Tests and Proofs*, volume 5668 of *LNCS*, pages 153–168. Springer, 2009.