Simulation Relations for Rich Acceptance Conditions

César Sánchez[1,2] *and* ***Julian Samborski-Forlese***[1]

[1]IMDEA Software Institute, Madrid, Spain
[2]Institute for Information Security, CSIC, Spain

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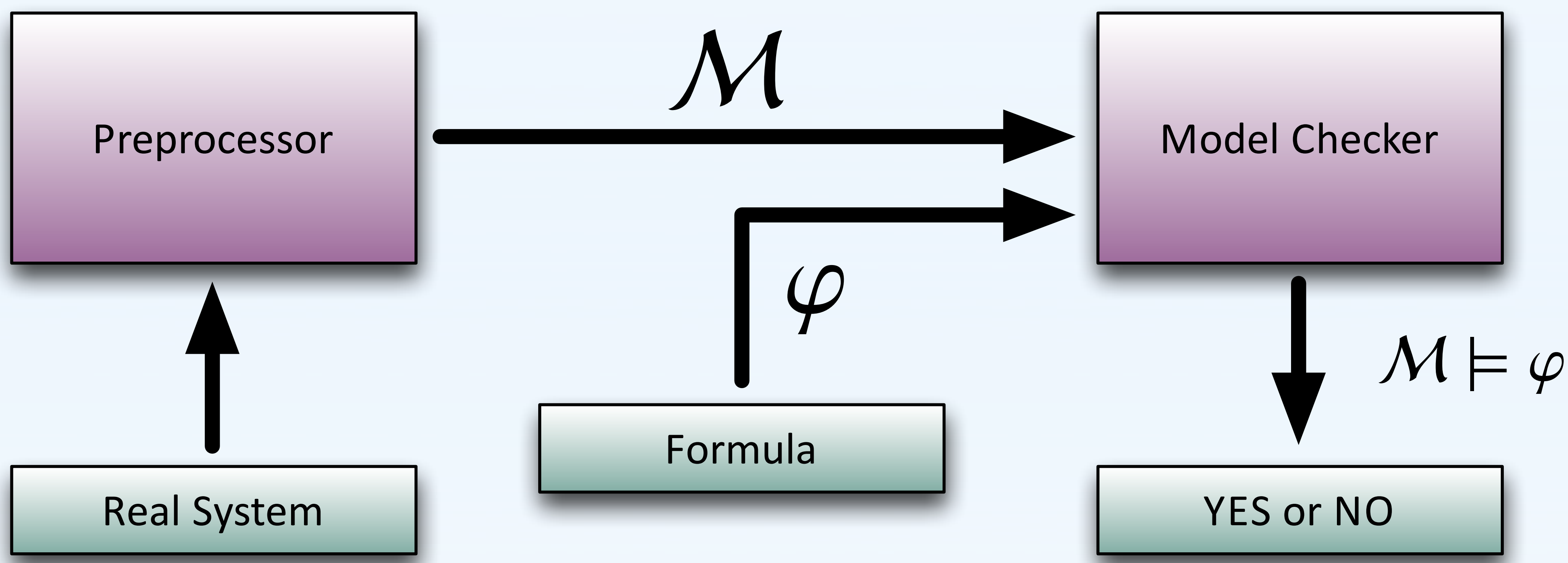
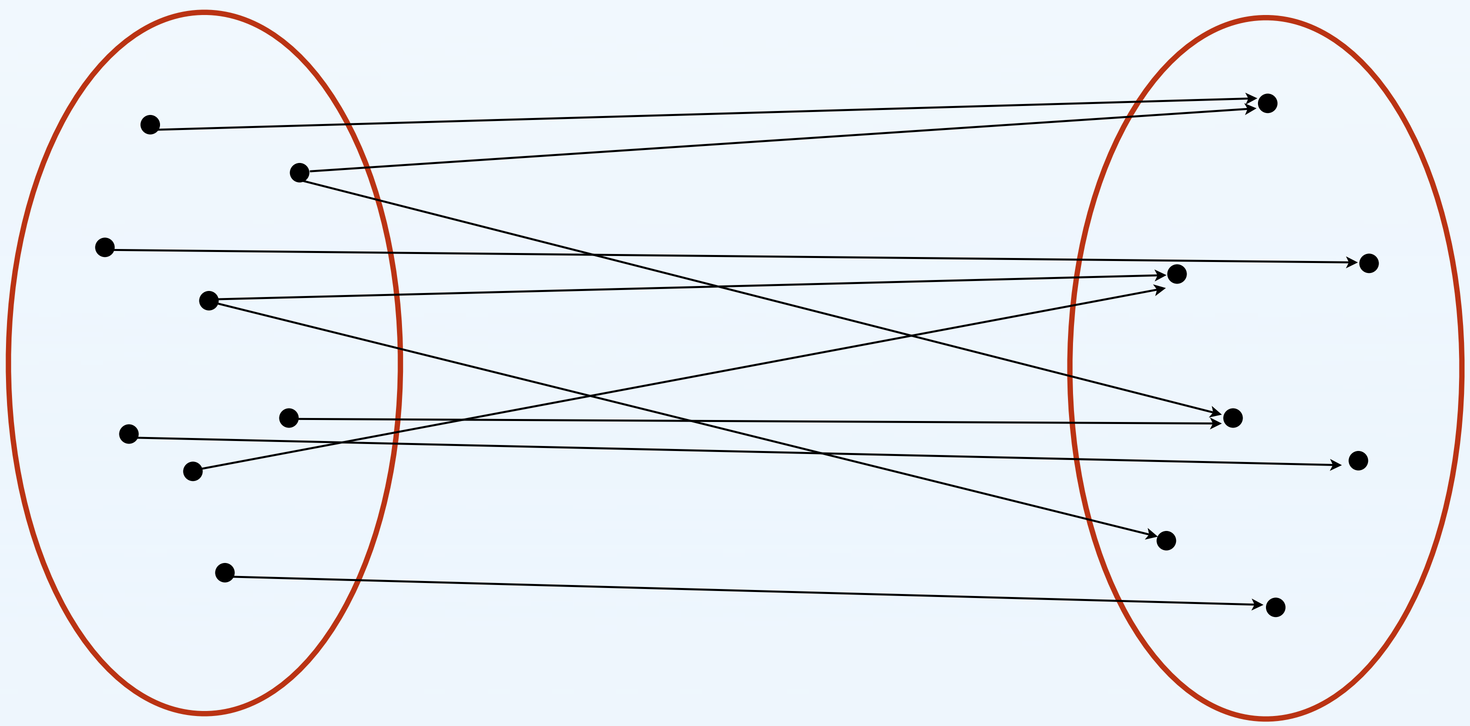
Quick Model Checking Intro



Antichain-based Model Checking [Raskin06]



LTL ⟶ **ABW** $\xrightarrow{\text{MH}}$ **NBW**

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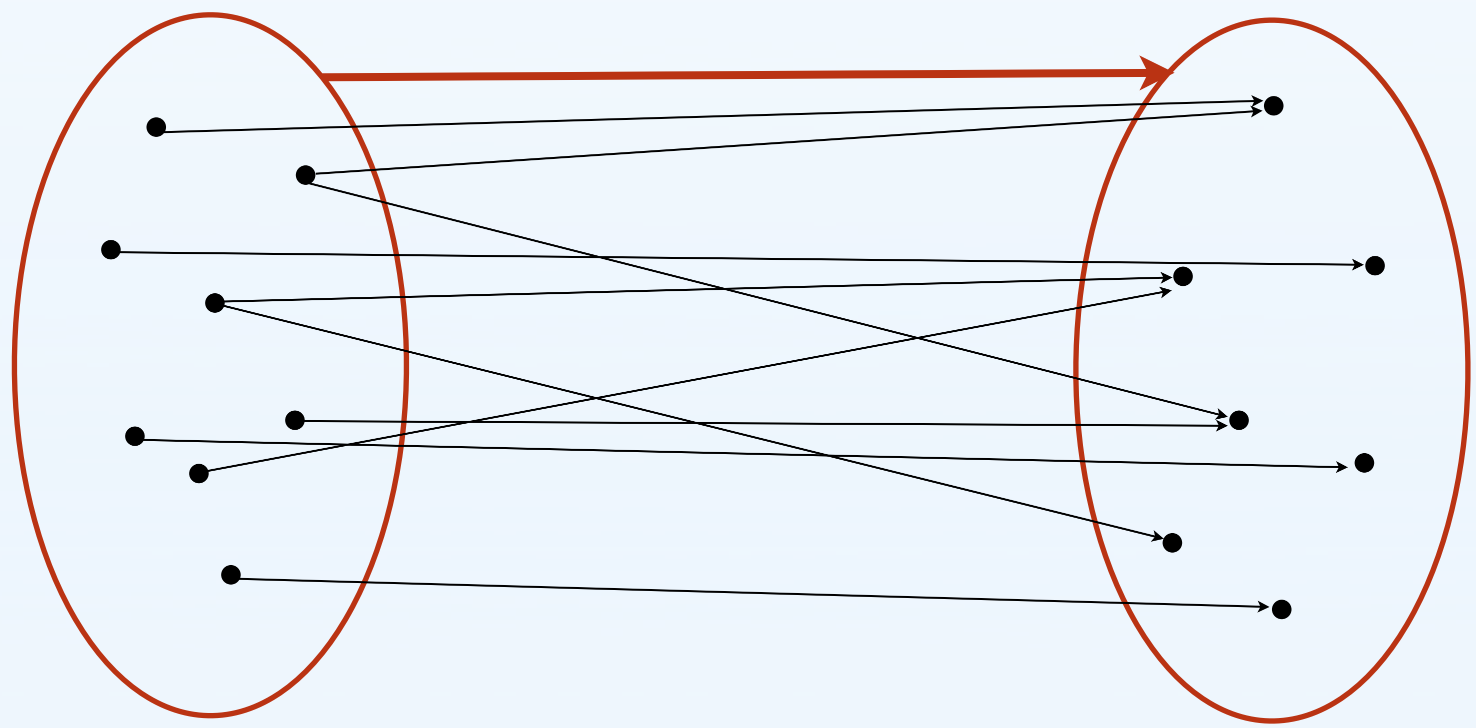

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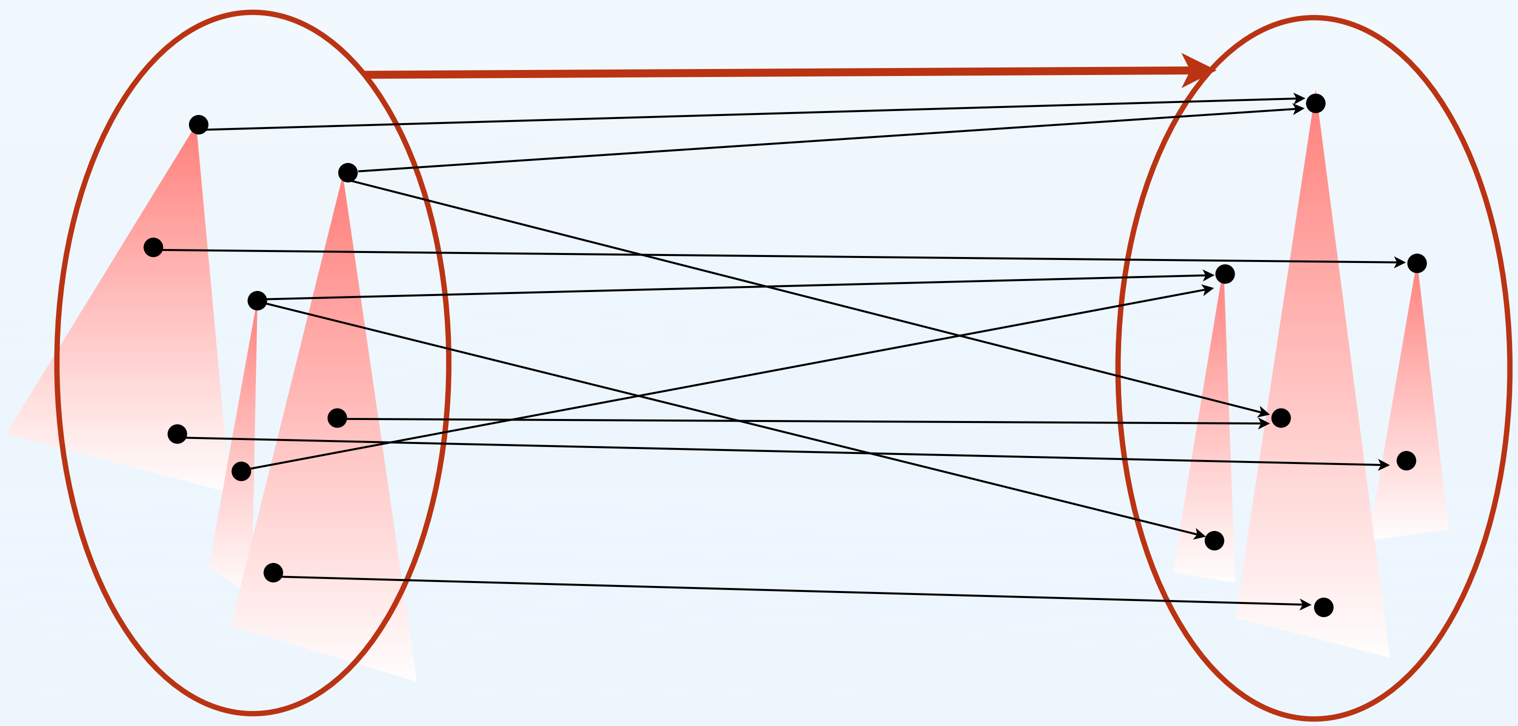

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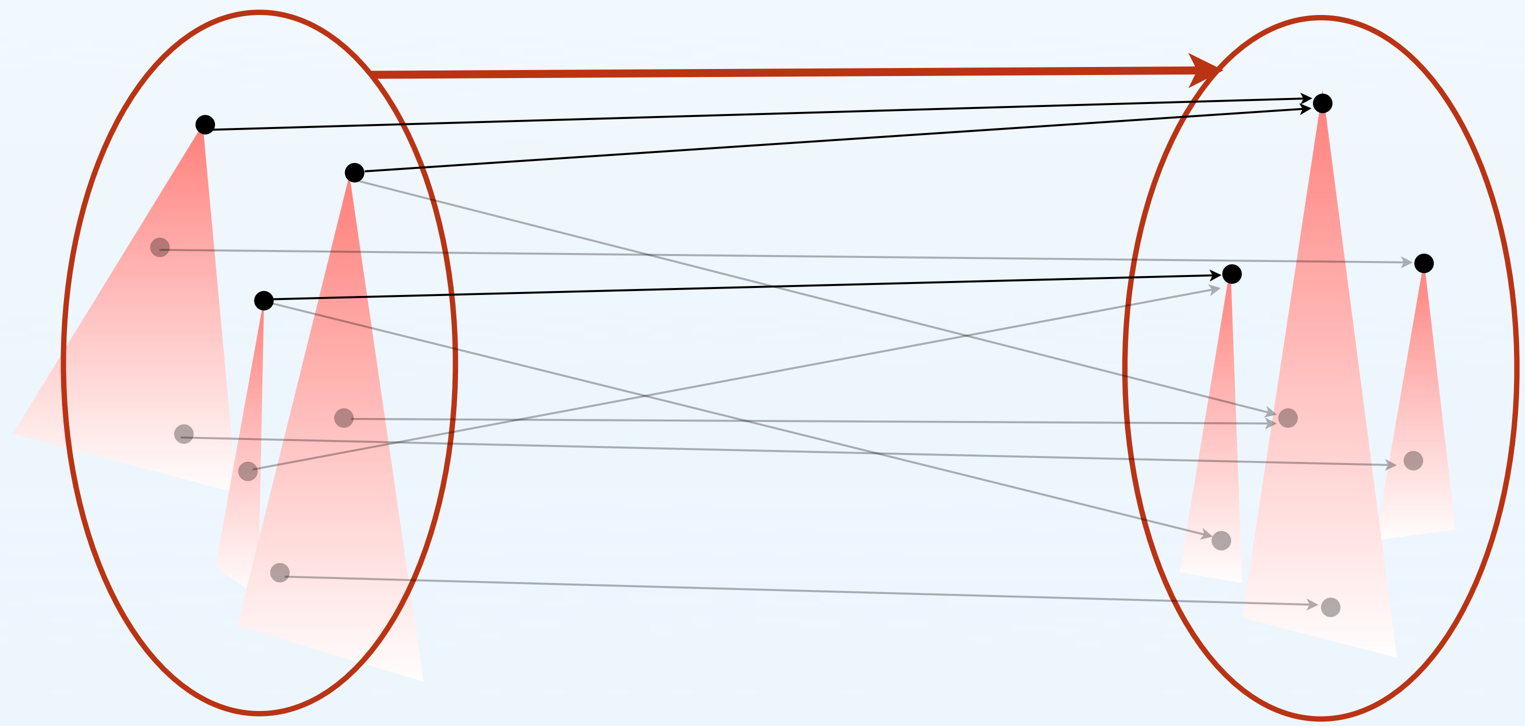

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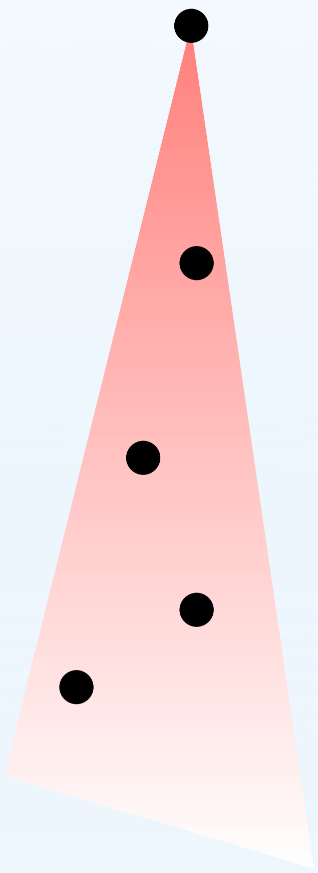

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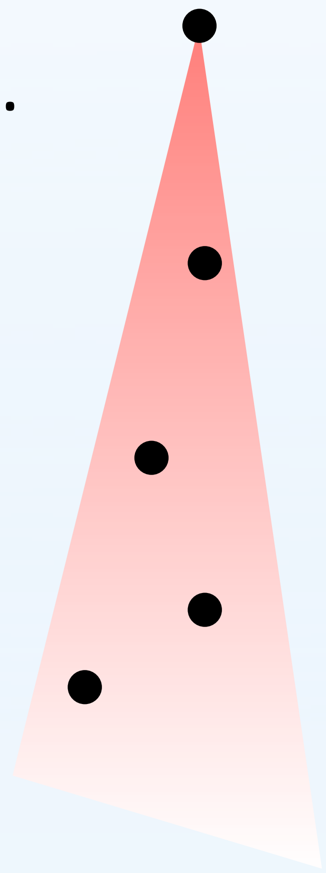

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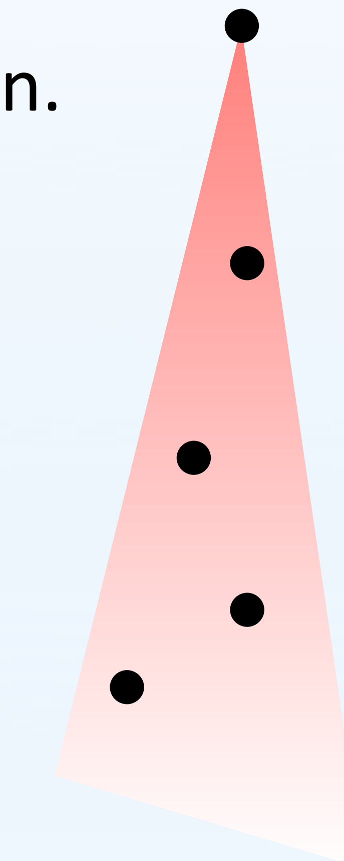

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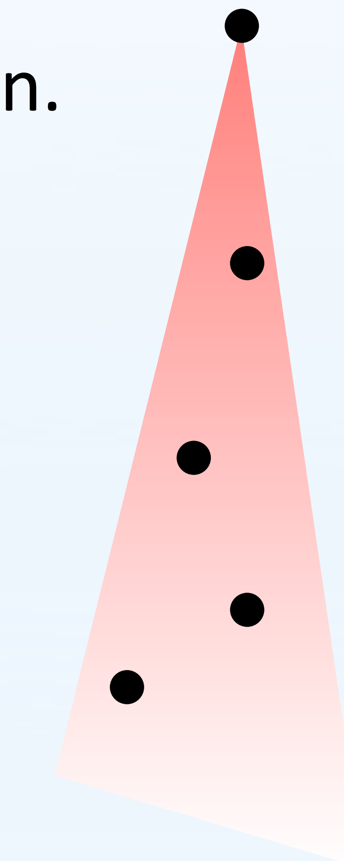

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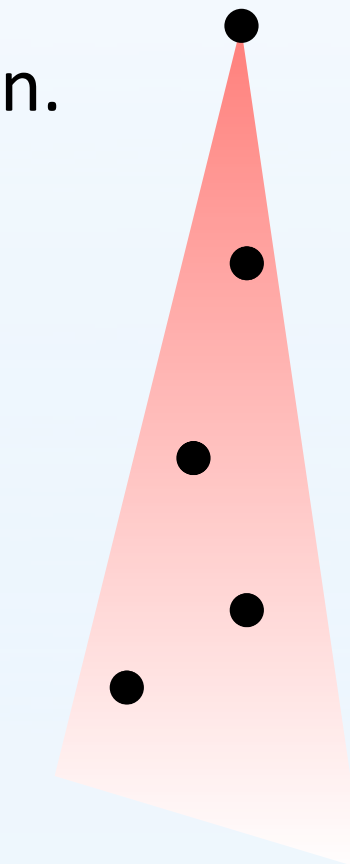

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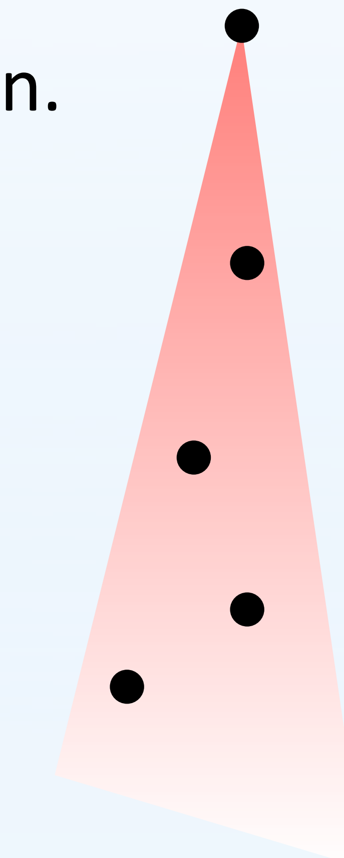

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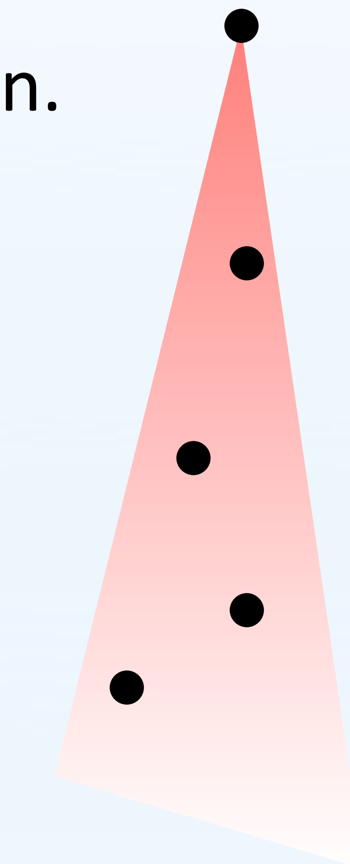

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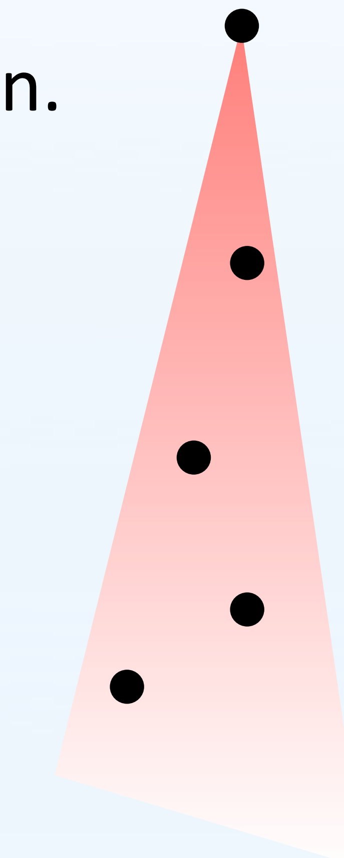

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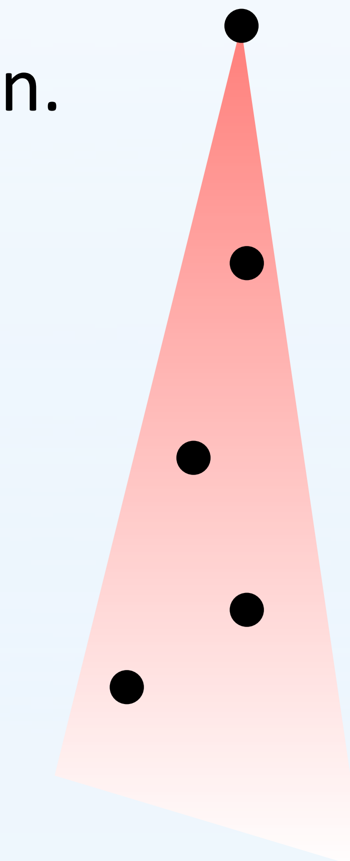

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- But, MH does not work for more complex logics.
- **Original antichain-based algorithms cannot be applied to model checking extensions of LTL.**

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Our Goal

Develop antichain-based algorithms for extensions of LTL

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  - Parity $\quad Q \rightarrow \{0..k\} \quad max(F(inf(\pi)))$ is even
  - Street⟨1⟩ $\quad (B, G) \quad$ if $inf(\pi) \cap B \neq \emptyset$ then $inf(\pi) \cap G \neq \emptyset$
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$\preceq_{\mathsf{f}}$

q_2

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q_2 forward simulates q_1

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$$\mathsf{BB}^*(M) = \mathsf{GFP}(\ \ X \cdot \mathsf{pre}^+(X) \cap F)$$

Language Emptiness

$$\mathcal{L}(\mathcal{N}) = \emptyset \text{ iff } \mathsf{BB}^* \cap I = \emptyset$$

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- **Symbolic Backward Repeated Reachability**

$$\widehat{\mathsf{BB}}^*(M) = \mathsf{GFP}(\lambda X \cdot \lceil F \rceil \ \sqcap \ \widehat{\mathsf{pre}}^+(X)) \qquad (\preceq_{\mathsf{f}})$$

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$$\mathcal{A} = (\Sigma, Q, \delta, I, F)$$

$$\delta_{\mathcal{N}} : Q_{\mathcal{N}} \to 2^{Q_{\mathcal{N}}}$$

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$$\mathbf{D.}\ \delta_{\mathcal{N}} = \begin{array}{l} \left\{ \langle (\ {}_1\ \varnothing)\ \sigma\ (\ {}_2\ \ {}_2 \setminus\ \ )\rangle \;\middle|\; {}_1 \overset{\sigma}{\rightsquigarrow}\ {}_2 \right\} \cup \\ \left\{ \langle (\ {}_1\ \ {}_1 \neq \varnothing)\ \sigma\ (\ {}_2\ \ {}_2 \setminus\ \ )\rangle \;\middle|\; {}_1 \overset{\sigma}{\rightsquigarrow}\ {}_2\ \ {}_1 \overset{\sigma}{\rightsquigarrow}\ {}_2\ \ {}_2 \subseteq\ {}_2 \right\} \end{array}$$

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Q1. *if q B then f(q) is e en.*

Q2. *if* $O \neq \emptyset$ *then* $ok = false$.

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I. M *is a minimal model of* $I_{\mathcal{N}}$
$O = \{q \in M \mid q \notin G \text{ and } f(q) \text{ is even}\}$ *and* $ok = \mathit{false}$.

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D1. $S' = \cup_{q \in S} M_q$,

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D2. $f'(p) \leq min\{f(q) \mid q \in pred(p) \setminus G\}$

D3. O' *is given as follows. Let* $p \in S' \setminus G$, *we have*

- *If* $ok = true$ *then* $p \in O'$ *iff* $f'(p)$ *is even.*
- *If* $ok = false$ *then* $p \in O'$ *iff* $f'(p) = f(q)$ *for some* $q \in (pred(p) \cap O)$.

Our Contribution

Definition (Streett Simulation Relation) *The Streett simulation relation* $\preceq$ *on* $\mathsf{S}(\mathscr{A}) \subseteq Q_N \times Q_N$ *is defined as* $(S_2, O_2, f_2, ok_2) \preceq (S_1, O_1, f_1, ok_1)$ *whenever:*

- **S1**. *for all* $q_2 \in S_2 \cap G$ *there is a* $q_1 \in S_1$ *with* $q_2 \ll q_1$.
- **S2**. *for all* $q_2 \in S_2 \cap \overline{G}$ *there is a* $q_1 \in S_1$ *with* $q_2 \ll q_1$ *and* $f_1(q_1) \leq f_2(q_2)$.
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Lemma *The Streett simulation relation* $\preceq$ *is a forward simulation on* $\mathsf{S}(\mathscr{A})$ *compatible with final states.*

Summary

- Antichains is a very cleaver model checking technique.
- Applied successfully to LTL model checking, outperforming traditional approaches.
- Showed the existence of simulation preorders on our more complex Streett construction.
- Similar results for our Rabin construction.

- Future guidelines:
  - Similar results for other interesting acceptance conditions (like Hesitant).
  - Implement antichains for RLTL (and possibly for PSL) and integrate into NuSMV.