Verification by abstraction and specialisation of constraint logic programs

John Gallagher[1,2]

[1] Roskilde University [2] IMDEA Software Institute, Madrid

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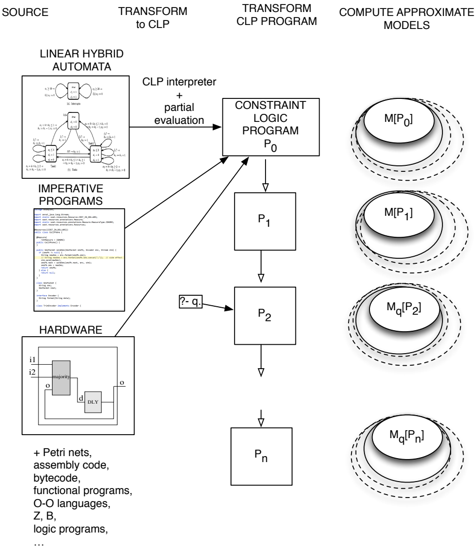
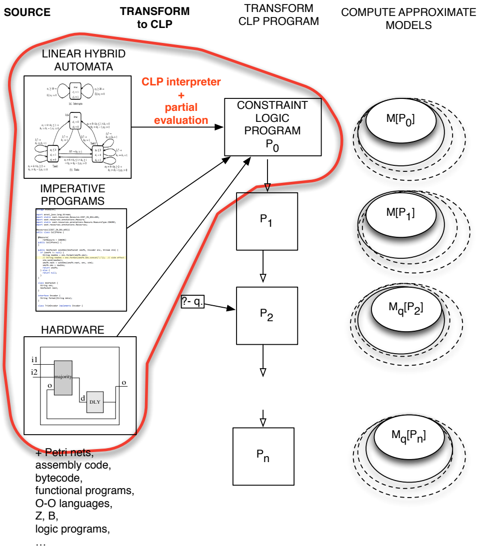
Map of the Talk



From Semantics to CLP



From Semantics to CLP interpreters

- Judgement

$$\frac{\alpha_1, \ldots, \alpha_n}{\alpha} \qquad \text{where } b$$

- CLP

$$\alpha \text{ :- } \alpha_1, \ldots, \alpha_n, b.$$

Note that the definitions of α_i, b can be “programmed" in CLP (cf. Manuel’s talk).

From Semantics to CLP interpreters: example

Current work: modelling the semantics of XC.

- Judgement

$$\frac{\langle S_1 \sigma \rangle \xrightarrow{L} \langle S_1' \sigma' \rangle}{\langle (S_1 \parallel S_2) \sigma \rangle \xrightarrow{L} \langle (S_1' \parallel S_2) \sigma' \rangle}$$

- Coq representation

```
ex_par_1_step : forall s1 s1' s2 st st' l r,
    exec s1 st l s1' st' r
-> exec (PAR s1 s2) st l (PAR s1' s2) st' r
```

- CLP representation

```
% ex_par_1_step
exec(par(S1, S2), St, L, par(S11, S2), St1,R) :-
    exec(S1, St, L, S11, St1,R).
```

CLP interpreters: defining a run

- Multi-step computation

$$\frac{}{\langle \text{skip } \sigma\rangle \rightarrow^* \langle \text{skip } \sigma\rangle}$$

$$\frac{\langle S_0\ \sigma_0\rangle \xrightarrow{\epsilon} \langle S_1\ \sigma_1\rangle \quad \langle S_1\ \sigma_1\rangle \rightarrow^* \langle S_2\ \sigma_2\rangle}{\langle S_0\ \sigma_0\rangle \rightarrow^* \langle S_2\ \sigma_2\rangle}$$

- CLP

```
run(skip,St,skip,St,0).
run(S,St,S2,St2,R) :-
    exec(S,St,emptyl,S1,St1,R1),
    run(S1,St1,S2,St2,R2),
```

Partial evaluation

- Experiments with offline partial evaluator LOGEN (M. Leuschel)
- The CLP interpreter is annotated to indicate
  - which calls are unfolded
  - a “filter" for each argument controlling generalisation and removal of static structures
- Essentially, everything is unfolded except
  - the recursive calls to “run"
  - the computations on dynamic values
- Program’s syntactic structure is removed

Example. XC program semantics instrumented with resource usage (e.g. energy) as final argument.

```
% factorial

function factnonrec(int n)
    int m=1;
    while(n > 0)
        m = m*n
        n = n-1
    return m
```

```
test7__0(A,B,C) :-
    runeval__2(B,A,1,1,D),
    C is 1+D.
runeval__2(A,B,C,D,E) :-
    B>0,
    F is B*C,
    G is B-D,
    runeval__2(A,G,F,D,H),
    E is 9+H.
runeval__2(cns(nat(A)),0,A,B,C) :-
    C is 4.
```

Structure filtering

“Flattening" transformation – removes redundant structure and retains only the dynamic values.
This is important to enable analysis of the partially evaluated program.

```
/*
runeval(stm(let(n,cns(nat(C)),let(m,cns(nat(D)),seq(ifnz(var(n),
seq(seq(asg(m,mul(var(n),var(m))),asg(n,sub(var(n),cns(nat(E))))),
while(var(n),seq(asg(m,mul(var(n),var(m))),
asg(n,sub(var(n),cns(nat(F))))))),skip),ret(var(m)))))),[],A,[],B) :-
runeval__3(A,F,C,D,E,B). */


runeval__3(A,F,C,D,E,B) :-
   runeval__4(A,F,C,D,C,E,G), B is 1+G.
```

Transition systems: CLP program encoding reachable states

transition(X,X')	$\leftarrow$	c_1(X,X').
transition(X,X')	$\leftarrow$	c_2(X,X').
$\cdots$	$\leftarrow$	$\cdots$
initState(X)	$\leftarrow$	c_{init}(X).
reach(X)	$\leftarrow$	initState(X).
reach(X')	$\leftarrow$	reach(X), transition(X,X').

The transition relation for a given system can be unfolded in the reach clauses.

$c_i(X, X')$ are constraints over some domain.

Generating assertions from semantics

- Judgement

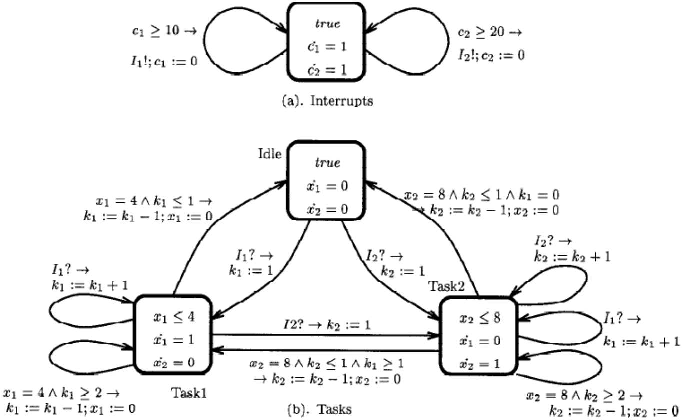
$$\frac{\sigma \not\models p}{\langle \text{assert } p\ \sigma \rangle \rightarrow \langle \text{error} \rangle}$$

$$\frac{\text{initState}\langle S\ St \rangle \quad \langle S\ \sigma \rangle \rightarrow^{*} \langle \text{error} \rangle}{\mathit{false}}$$

- CLP representation

```
exec(assert(P),St, error) :- ¬P.
false :- init(S,St), exec(S,St,error).
```

Example: A task scheduler [Halbwachs et al. 94]



Example Transition for Scheduler

Sample transition of Scheduler.

```
transition((J, L, N, P, R, S, G),(A, B, C, D, E, F, 0)) :-
    G<H,
    1*I=1*J+1*(H-G),
    1*K=1*L+1*(H-G),
    1*M=1*N+0*(H-G),
    1*O=1*P+0*(H-G),
    1*Q=1*R+0*(H-G),
    1*_=1*S+0*(H-G),
    K>=20, A=I, B=0,
    C=M, D=O, E=Q,
    F=1.
```

Semantics for termination

- Binary clause semantics (Codish et al., 1999, derived from a more general “resolvent" semantics for logic programs).
- Binary clauses can be derived from a CLP meta-program by partial evaluation (Gallagher, LOPSTR’03)

```
bin(rev([X|Xs],Zs),Q) :-
  bin(rev(Xs,Ys),Q).
bin(rev([X|Xs],Zs),Q) :-
  rev(Xs,Ys), bin(app(Ys,[X],Zs),Q).
bin(app([X|Xs],Ys,[X|Zs]),Q) :-
  bin(app(Xs,Ys,Zs),Q).
bin(rev(X,Y),rev(X,Y)) :- true.
bin(app(X,Y,Z),app(X,Y,Z)) :- true.
```

Good partial evaluations

To be useful for analysis, the partially evaluated CLP program should:

- be of the same size order as the original program,
- predicates correspond (more or less) to program points
- remove all the source program syntax.

Is this always possible?

Big-step vs. small-step semantics

The form of the semantic judgements determines the form of the CLP program.

- Big-step semantics generally makes it easier to obtain a “good" partial evaluation.
- Small-step semantics produces programs that are “transition systems" and are “easier to analyse"; but . . .
- For recursive programs, small-step semantics requires a stack to be represented explicitly in the CLP program
- For big-step semantics, the stack is implicit in the CLP semantics.
- Compound data structures, heap, etc. need careful consideration in order to get an analysable CLP program.

Big-step vs. small-step semantics - cont'd

big-step

```
proc :-
  stmt1,
  stmt2,
  .  .  .
  stmtn.
```

small-step

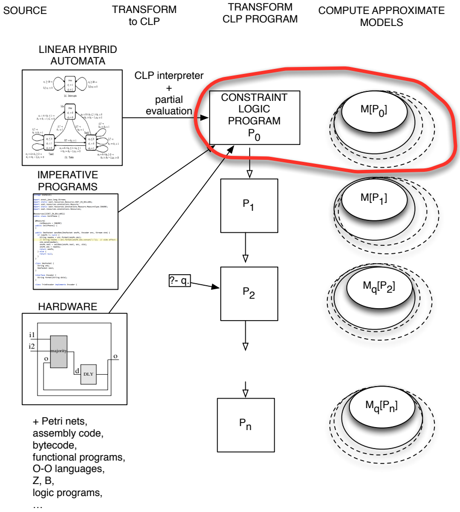
```
proc :-
  stmt1.
stm1 :-
  stmt2.
  .  .  .
stmtn-1 :-
  stmtn.
stmtn :-
  .  .  .
```

Semantics to CLP - Summary

CLP programs (should be) derived systematically from semantics:

- CLP representation of semantic judgements (e.g. operational semantics, proof rules)
- Semantics possibly instrumented or enhanced with traces, etc.
- Partially evaluate semantics wrt a fixed program to get a CLP program
- Filter out the syntactic structures from the interpreter, leaving a CLP program over the domain of the program

Computing (approximate) models of CLP programs



CLP model semantics

Here we focus on the model semantics (in contrast to the proof semantics). A model is a set of constrained facts.
The “immediate consequence" operator for a CLP program (a generalisation of the standard T_P function).

$$T_P^{\mathcal{C}}(I) = \left\{ A \leftarrow \mathcal{C} \;\middle|\; \begin{array}{l} A \leftarrow B_1, \ldots, B_n, D \in P \\ \{A_1 \leftarrow \mathcal{C}_1, \ldots, A_n \leftarrow \mathcal{C}_n\} \in I \\ \exists\, \theta \text{ such that} \\ mgu((B_1, \ldots, B_n), (A_1, \ldots, A_n)) = \theta \\ \mathcal{C}' = \bigcup_{i=1,\ldots,n} \{\mathcal{C}_i\theta\} \cup D \\ \mathrm{SAT}(C') \\ \mathcal{C} = \mathrm{proj}_{Var(A)}(\mathcal{C}') \end{array} \right\}$$

$$M^{\mathcal{C}}[\![P]\!] = \mathrm{lfp}(T_P^{\mathcal{C}})$$

Checking program properties

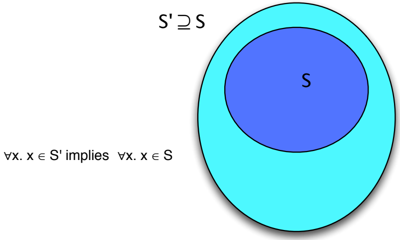
- The minimal model is equivalent to the set of derivable facts of the program.
- So we can check whether $P \models A$ either
  - by checking whether $A \in M[\![P]\!]$
  - or, by running A as a query to P (using a complete proof rule such as tabling (cf. Manuel's talk).
- Other semantics (e.g. greatest fixpoints) are also relevant to other problems (see later in talk).

Computing Fixpoints

- The minimal model is computed as the least fixed point of the immediate consequences function $T_P^{\mathcal{C}}$.
- This is the limit of the Kleene sequence $\emptyset, T_P^{\mathcal{C}}(\emptyset), T_P^{\mathcal{C}}(T_P^{\mathcal{C}}(\emptyset)), \ldots$.
- In general this is not a finite sequence – hence approximation is required.
  - either in the model computation (bottom-up) or in the computation (top-down).

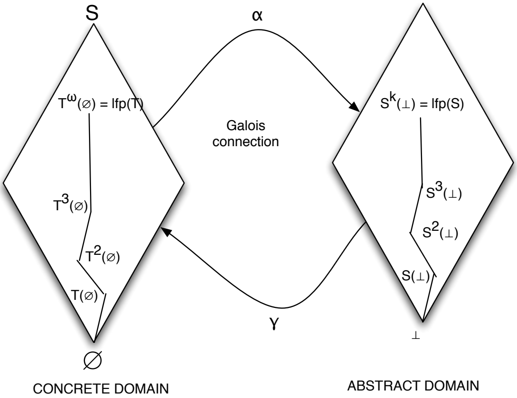
Proofs by Approximation

The core of verification using static analysis is proof by approximation.
Over-approximation gives us sufficient conditions for proving universal formulas over some infinite set.



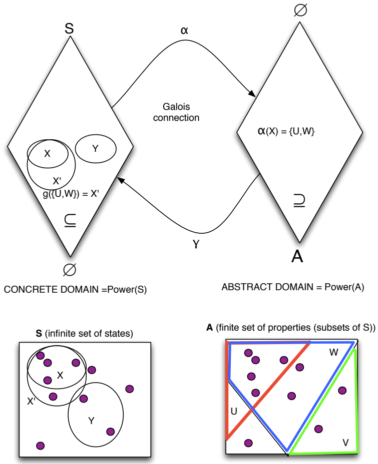
cf. Manuel’s talk and references for a full account of verification by analysis.

Abstract interpretation of CLP in one picture.



Safety condition: $T \circ \gamma \subseteq \gamma \circ S$

A property-based abstraction is an abstract interpretation.



Tree automata abstractions

Operations on a token ring (with any number of processes)
(example from Podelski & Charatonik).

```
gen([0,1]).
gen([0 | X]) ← gen(X).
trans(X,Y) ← trans1(X,Y).
trans([1 |X],[0|Y]) ← trans2(X,Y).
trans1([0,1|T],[1,0 |T]).
trans1([H|T],[H|T1]) ← trans1(T,T1).
trans2([0],[1]).
trans2([H|T],[H|T1]) ← trans2(T,T1).
reachable(X) ← gen(X).
reachable(X) ← reachable(Y), trans(Y,X).
```

What are the possible answers for reachable(X)? Can X be a list containing more than one '1'?

```
gen([0,1]).
gen([0,0,1]) .
gen([0,0,0,...,1]).
....
```

Intended reachable states
reachable([0,0,...,1,...0,0])
(lists with exactly one 1)

A proof of safety can be found by approximating the infinite model of this program by a tree automaton (regular type inference, cf. Manuel's talk).

Abstraction by Intervals, polyhedra, $\cdots$

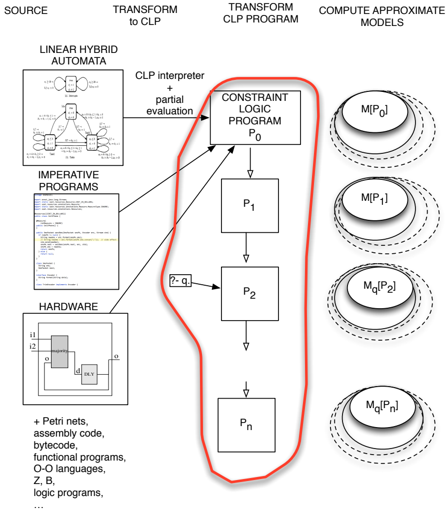
- Example.
  ```
  applen(X,Y,Z) :- X=0, Y=Z, Y>=0.
  applen(X,Y,Z) :- applen(X1,Y,Z1), X = X1+1, Z = Z1+1.
  revlen(X,Y) :- X=0,Y=0.
  revlen(X,Y) :- revlen(X1,Z),applen(Z,U,Y),X=X1+1, U=1.
  false :- revlen(X,Y), X>Y.
  false :- revlen(X,Y), X<Y.
  ```
- Approximation by convex hulls gives:
  ```
  applen(X,Y,Z) :- X+Y=Z.
  revlen(X,Y) :- X=Y
  ```

Note that widenings are used in these abstract domains, since they are not of finite height.

Summary - computing approximate models

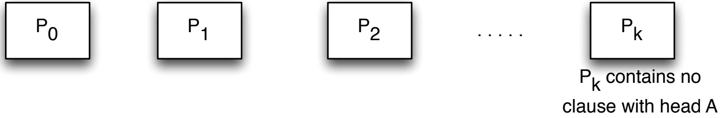
- Abstract interpretation provides a systematic framework for generating sound approximations of the models of CLP programs.
- A great variety of useful abstract domains has been developed.
- Abstraction can be combined with refinement heuristics to improve the precision of abstractions.
- Generic optimisation of fixpoint computation has been studied (program SCCs, worklists, semi-naive evaluations,...).

Proof by CLP transformation

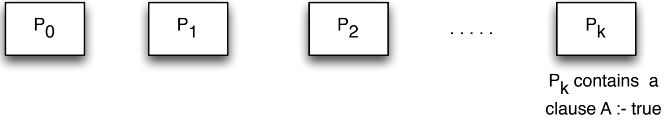


Proof by CLP transformation (overall idea)

Given a CLP program P_0, say we wish to show that some atom *A* is not a consequence.



Suppose we wish to prove that *A* is a consequence.



Transformation rules preserve the model (wrt to some specified predicates).

The MAP system (Pettorossi, Proietti et al.)

- The MAP system is an automatic program transformation system that automatically proves properties of CLP programs.
- Compares favourably with ARMC, HSF(C) and TRACER (see De Angelis et al. PEPM 2013)

Abstraction techniques related to abstract interpretation are used during the transformations.

The PRO-B system (Leuschel)

- The Pro-B system is an automatic program specialisation system that automatically proves properties of CLP programs.
- It is now being commercialised.
- The main proof technique is program specialisation - again aiming to make program properties explicit.

Query-answer transformations

- A generalisation of “magic set" transformations for Datalog
- For each predicate p, define two predicates p_{ans} and p_{query}.
- Given a program P and query Q, derive a program P_Q.
- $P \models Q$ iff $P_Q \models Q_{ans}$.

Query-answer transformation allows computation tree semantics to be simulated by model semantics. (The p_{query} predicates represent calls in the computation tree).

Proof for transformation - Summary

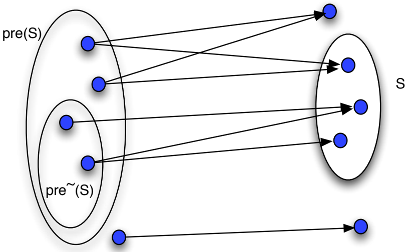
- Model-preserving transformations are applied
- Proof is obtained when the required property becomes explicit in the transformed program.
- Specialisation wrt a query is a very useful form of transformation – achieved by query-answer transforms, or by various specialisation algorithms.

Final topic: Abstract model checking of CLP transition programs

- We start with a CLP representation of a transition system.
- Each transition is a clause of form $\mathit{transition}(X, X') :- c(X, X')$, also represented as $\bar{X} \stackrel{c(\bar{X},\bar{X}')}{\longrightarrow} \bar{X}'$.
- $c(X, X')$ is a constraint over some constraint domain.

pre and $\widetilde{pre}$ functions

From a transition relation, compute functions $pre : 2^S \to 2^S$, $\widetilde{pre} : 2^S \to 2^S$.



- $pre(Z)$: the set of possible predecessors of set of states Z.
- $\widetilde{pre}(Z)$: the set of definite predecessors of set of states Z.

pre and $\widetilde{pre}$ expressed using constraint operations

A constraint $c(\bar{X})$ stands for the set of states satisfying $c(\bar{X})$.

$$
\begin{aligned}
pre(c'(\bar{y})) &= \bigvee\{\exists \bar{y}(c'(\bar{y}) \wedge c(\bar{x},\bar{y})) \mid \bar{x} \xrightarrow{c(\bar{x},\bar{y})} \bar{y} \text{ is a transition}\}\\
\widetilde{pre}(c'(\bar{y})) &= \neg(pre(\neg c'(\bar{y})))
\end{aligned}
$$

We assume that the constraint solver has a projection ($\exists$-elimination) operation and is closed under boolean operations.

Checking CTL properties

Define a function $[\![\phi]\!]$ returning the set of states where ϕ holds.
Compositional definition:

$$
\begin{array}{ll}
[\![p]\!] & = \mathsf{states}(p) \\
[\![EF\phi]\!] & = \mathrm{lfp}.\lambda Z.([\![\phi]\!] \cup pre(Z)) \\
[\![AG\phi]\!] & = \mathrm{gfp}.\lambda Z.([\![\phi]\!] \cap \widetilde{pre}(Z)) \\
\ldots &
\end{array}
$$

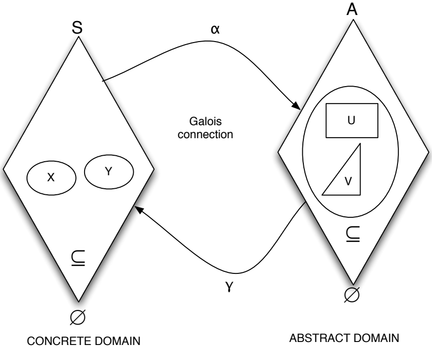
where states(p) is the set of states where proposition p holds (i.e. a constraint).

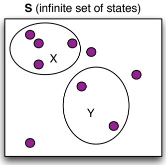
Model checking ϕ:

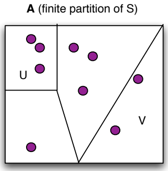
1. Evaluate $[\![\phi]\!]$.
2. Check that $I \subseteq [\![\phi]\!]$, where I is the set of initial states.

Equivalently, check that $I \cap [\![\neg\phi]\!] = \emptyset$.

Galois connection for partition abstraction







Galois connection implemented using constraint operations

Assume that the elements of the partition are given by constraints. Let c_d be the constraint defining the partition element d.

$$\alpha(c) = \{d \in A \mid \mathrm{SAT}(c_d \wedge c)\}$$
$$\gamma(V) = \bigvee\{c_d \mid d \in V\}$$

- *SAT* can be implemented by an SMT solver. We used Yices (http://yices.csl.sri.com/) interfaced to Prolog.

Abstraction of functions

Given a function

$$f : 2^S \rightarrow 2^S$$

on the concrete domain, the most precise approximation of f in the abstract domain is

$$\alpha \circ f \circ \gamma : 2^A \rightarrow 2^A.$$

Applying this construction to the function $[\![.]\!]$, obtain a function $[\![\phi]\!]^a$.

$$\begin{array}{ll} [\![p]\!]^a & = (\alpha \circ \mathsf{states})(p) \\ [\![EF\phi]\!]^a & = \mathsf{lfp}.\lambda Z.([\![\phi]\!]^a \cup (\alpha \circ pre \circ \gamma)(Z)) \\ [\![AG\phi]\!]^a & = \mathsf{gfp}.\lambda Z.([\![\phi]\!]^a \cap (\alpha \circ \widetilde{pre} \circ \gamma)(Z)) \\ \ldots & \end{array}$$

Computation of $[\![\phi]\!]^a$ terminates. It can be shown that for all ϕ,

$$[\![\phi]\!] \subseteq \gamma([\![\phi]\!]^a)$$

.

Abstract Model Checking of ϕ

1. Compute $[\![\neg\phi]\!]^a$.
2. Check that $I \cap \gamma([\![\neg\phi]\!]^a) = \emptyset$.
3. This implies that $I \cap [\![\neg\phi]\!] = \emptyset$, since $\gamma([\![\neg\phi]\!]^a) \supseteq [\![\neg\phi]\!]$.

Some Experiments on Linear Hybrid Automata

Arbitrary CTL formulas can be checked (not just A-formulas as in standard abstract model checking).

System	*Property*	A	Δ	*secs.*
Water Monitor	$AF(W \geq 10)$	5	4	0.02
	$AG(0 \leq W \wedge W \leq 12)$	5	4	0.01
	$AF(AG(1 \leq W \wedge W \leq 12))$	5	4	0.02
	$AG(W = 10 \rightarrow AF(W < 10 \vee W > 10))$	10	4	0.05
	$AG(AG(AG(AG(AG(0 \leq W \wedge W \leq 12)))))$	5	4	0.02
	$EF(W = 10)$	10	4	0.01
	$EU(W < 12, AU(W < 12, W \geq 12))$	7	4	0.04
Task Sched.	$EF(K2 = 1)$	18	12	0.53
	$AG(K2 > 0 \rightarrow AF(K2 = 0))$	18	12	0.30
	$AG(K2 \leq 1)$	18	12	0.04

CLP-based verification: Some directions

- Systematic generation of CLP program from semantics
- Refinement techniques for arbitrary abstract domains (not just predicate abstractions)
- Widening in predicate refinement
- Representation and abstraction of memory, heap, stack, etc.
- Program transformation vs. abstraction - understand the connections better.

THE END