Interpolant Strength in Model Checking

Based on CAV'12 work

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Outline

1. Background
   - Interpolation for Model Checking
   - Labeled Interpolation Systems

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   - Path Interpolation: Requirements and Constraints

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    - System model vs behavioral property specification

Symbolic Model Checking

Hardware and Software Verification

- Model Checking
  - System model vs behavioral property specification
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  - Problem solving by means of reasoning engines (SAT solvers)

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Property-based overapproximation

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Interpolation

Open Issues and Contribution

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- Various interpolation systems [P97,McM04,DKPW10]
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$\Rightarrow$ Formalization of requirements for simultaneous abstraction, path interpolation

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⇒ Formalization of requirements for simultaneous abstraction, path interpolation

⇒ Identification of subset of interpolation systems satisfying requirements

Interpolation [Craig57,McM03]
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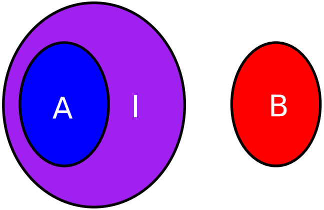

Interpolant Strength

Applications to Symbolic Model Checking

- I_1 stronger than I_2 $\qquad I_1 \rightarrow I_2$

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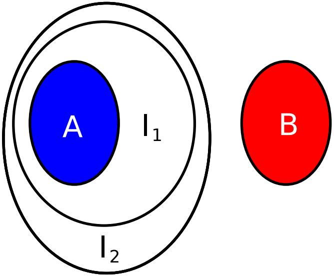

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  - Computation of I from proof structure

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SAT

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Resolution System
Background

- Literal $\quad p \quad \overline{p}$
- Clause $\quad p \lor \overline{q} \lor r \lor \ldots \rightsquigarrow p\overline{q}r\ldots$ $\quad$ Empty clause $\quad \bot$
- Input formula $\quad (p \lor q) \land (r \lor \overline{p}) \land \ldots \rightsquigarrow \{pq, r\overline{p}, \ldots\}$

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- Resolution rule

$$\frac{pC \qquad \overline{p}D}{CD}\, p$$

Antecedents: $pC\ \overline{p}D$ Resolvent: CD Pivot: p

Resolution Proofs

Resolution System

- Resolution proof of unsatisfiability of a set of clauses S

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$$
\cfrac{\cfrac{\cfrac{p\overline{q} \qquad \overline{pr}}{\overline{qr}}\,p \qquad r}{\overline{q}}\,r \qquad q}{\perp}\,q
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- Strength comparison can be reduced to labeling comparison

- Labeling L for $A \wedge B$

Labeling

Labeled Interpolation Systems

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- $A = (\overset{a}{\overline{p}} \vee \overset{?}{\overline{q}}) \wedge (\overset{a}{p} \vee \overset{?}{\overline{q}})$ $\quad$ $B = (\overset{?}{q} \vee \overset{b}{\overline{r}}) \wedge (\overset{?}{q} \vee \overset{b}{r})$

Labeling Lattice [DKPW10]

Labeled Interpolation Systems

- $b \preceq ab \preceq a \qquad \rightsquigarrow \qquad (\alpha_1, \overset{L_1}{\ldots}, \alpha_n) \preceq (\beta_1, \overset{L_2}{\ldots}, \beta_n)$

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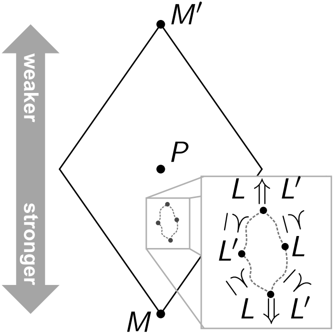

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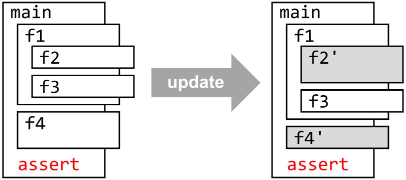
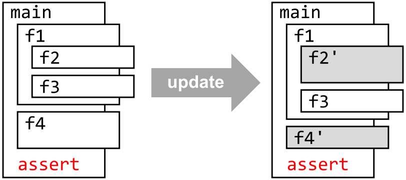
Software Upgrade Checking
Simultaneous Abstraction



- Program safe

Software Upgrade Checking

Simultaneous Abstraction



- Program safe $\phi_{main} \wedge \phi_{f1} \wedge \phi_{f2} \wedge \phi_{f3} \wedge \phi_{f4}$ UNSAT

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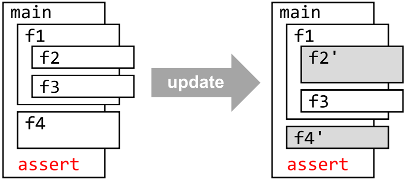

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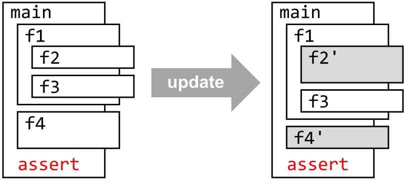

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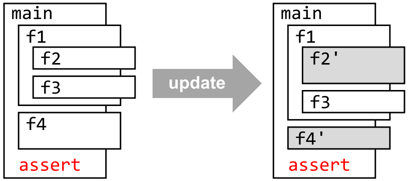

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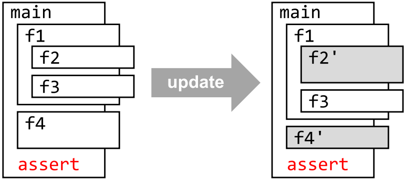

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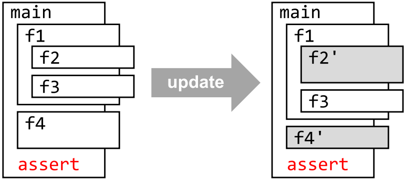

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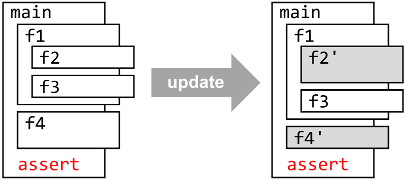

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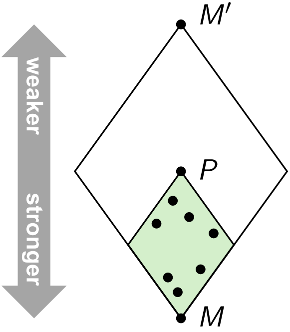

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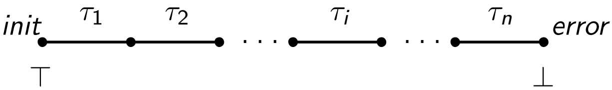

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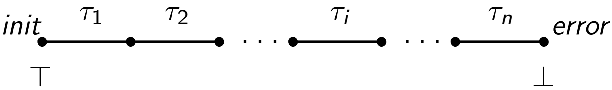

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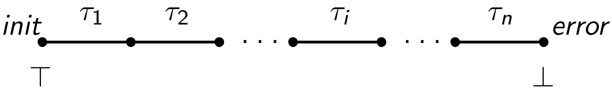

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  $\top \wedge \tau_1 \rightarrow I_1 \qquad I_i \wedge \tau_{i+1} \rightarrow I_{i+1} \qquad I_{n-1} \wedge \tau_n \rightarrow \bot$

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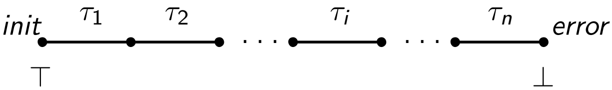

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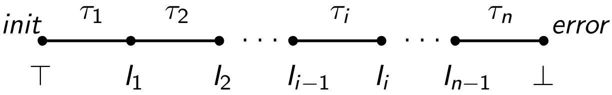

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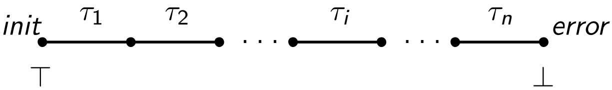
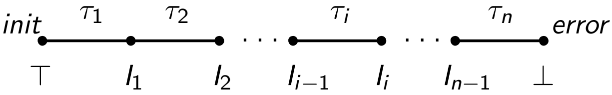


- Extract interpolants

$\top \wedge \tau_1 \rightarrow I_1$ $\quad I_i \wedge \tau_{i+1} \rightarrow I_{i+1}$ $\quad I_{n-1} \wedge \tau_n \rightarrow \bot$



- Requirement: $\tau_1 \to I_1 \qquad I_i \wedge \tau_{i+1} \to I_{i+1} \qquad I_{n-1} \wedge \tau_n \to \bot$

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- Satisfied for: $L_1 \preceq \ldots \preceq L_n$

- Requirement: $\tau_1 \rightarrow I_1 \qquad I_i \wedge \tau_{i+1} \rightarrow I_{i+1} \qquad I_{n-1} \wedge \tau_n \rightarrow \perp$
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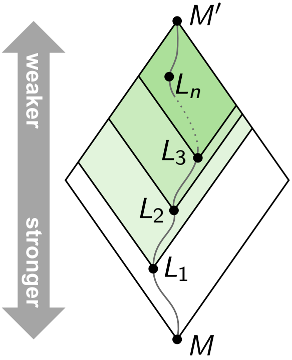

Outline

1. Background
   - Interpolation for Model Checking
   - Labeled Interpolation Systems
2. Contribution
   - Interpolant Strength in Model Checking
   - Simultaneous Abstraction: Requirements and Constraints
   - Path Interpolation: Requirements and Constraints
3. Conclusions

Summary

- Interpolant strength in symbolic model checking

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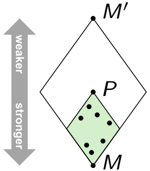
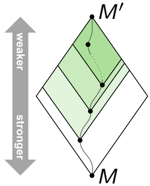


Sim. Abstraction



Path Interpolation

Ongoing Work

- Necessary and sufficient conditions for labeled interpolation systems

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  - Path interpolation, simultaneous abstraction, state-transition interpolation, tree interpolation

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Thanks for your attention!

`verify.inf.usi.ch`

Reference

S.F. Rollini, O. Sery and N. Sharygina
Leveraging Interpolant Strength in Model Checking.
CAV 2012.

Example

Labeled Interpolation Systems

- $A = \{p\overline{q}, r\} \qquad B = \{\overline{pr}, q\}$

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$$\dfrac{\dfrac{\dfrac{p\overline{q} \qquad \overline{pr}}{\overline{qr}} \qquad r}{\overline{q}} \qquad q}{\bot\,[(p \vee \overline{q}) \wedge r]}$$

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- $(b, b, b) \preceq (a, a, a) \quad \Longrightarrow \quad (p \vee \overline{q}) \wedge r \rightarrow (p \wedge r) \vee \overline{q}$

Transition Relation Approximation

Simultaneous Abstraction

- BMC: iterative analysis k-length traces

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  - Initial states S, abstract transition relation $\hat{T}$, error states E

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